On the Theoretical Derivation of the Normal Distribution for Psychological Phenomena

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ABSTRACT

Though research has indicated that distributions of certain psychological phenomena such as intelligence (as it is assessed on various intelligence tests) are approximately normal as a rule, insufficient attention has been paid to the importance of theoretically deriving the normal distribution for psychological phenomena from first principles. If one accomplishes this task in a manner similar to that found in statistical mechanics and thermal physics (and I am not aware of another plausible theoretical derivation), the premise is made that randomness lies at the foundation of the psychological phenomena so described. These phenomena are in their very nature indisputably characterized by some degree of order.

TEXT

If one is to derive theoretically the normal distribution for psychological phenomena from first principles, it appears that this derivation is analogous to that found in statistical mechanics and thermal physics for the description of physical systems composed of many distinguishable particles. In statistical mechanics and thermal physics, the first principles that allow for the derivation of the Gaussian distribution for the description of a closed physical system composed of many, distinguishable particles are: (1) the existence of a closed physical system; (2) what is called the fundamental assumption of statistical mechanics and thermal physics; and (3) an ensemble of physical systems all constructed like the actual closed physical system of interest except that each system is in exactly one of the accessible stationary quantum states for the closed system of interest. This "ensemble is an intellectual construction that represents at one time the properties of the actual system as they develop in the course of time" (Kittel, 1969, p. 32).

An isolated, or closed, physical system is a physical system in which the energy, the number of particles, and the volume remain constant over time. When all observable quantities of a physical system, including the energy, are independent of the time, the system is said to be in a stationary quantum state. The fundamental assumption of statistical mechanics and thermal physics is that an actual isolated system is equally likely to be in any accessible stationary quantum state (Kittel, 1969). This means that randomness is at the heart of the fundamental assurntpion. There is assumed to be enough leeway in the specification of the actual system so that, although we are concerned with the stationary state of the actual system as a result of the actual system being closed, the fundamental assumption is applicable. In conjunction with the fundamental assumption, the use of a representative ensemble allows for the calculation of average values of observable quantities for this group of systems at a particular time. These average values correspond to time average values of the same physical quantities that are predicted and experimentally determined for the actual closed physical system of interest when the system is repeatedly measured over time. The repeated measurements allow for the relaxation time characteristic of the system and are otherwise random in manner. It is to be emphasized that the physicist's imagination is the basis for the representative ensemble of physical systems like the actual system of interest and that the use of the representative ensemble has been verified by experiment.

That an isolated physical system composed of distinguishable, but similar, particles is equally likely to be in any accessible stationary quantum state may most reasonably be accomplished by assuming that each particle of the system undergoes independent selection with regard to the values of the quantum numbers needed to specify completely the state of the particle and that the particle is equally likely to have any of the accessible values of these numbers. Randomness is also at the heart of the assumption of the equal likelihood that each distinguishable particle in the system has any of the accessible values of the quantum numbers needed to specify its state completely. Consider an isolated physical system composed of many distinguishable, but similar, particles to which the fundamental assumption is applied. With the use of a representative ensemble, it can be mathematically derived that the Gaussian, or normal, distribution (or a multivariate normal distribution) is generally a very close approximation to the distribution found where the number of accessible stationary quantum states for the actual physical system is a function of the number of particles in a particular state.

It appears that in the use of statistics based on the normal distribution in the study of psychological phenomena the first principles of (1) an isolated system, (2) an assumption analogous to the fundamental assumption, and (3) a representative ensemble of systems are needed for the theoretical derivation of the normal distribution for such phenomena. As in physics, such theoretical considerations are necessary to ensure the scientific usefulness of any experiment in which such statistics are used. Without these considerations (i.e., if the applicability of the normal distribution to the psychological phenomenon studied is based solely on experience), the presumption that an independent test of theoretical propositions concerning, or involving, the normal distribution of the psychological phenomenon can be conducted by submitting them to empirical test would be suspect. Any "predictions" derived from these theoretical propositions would actually be fundamentally a posteriori and thus riot predictive in nature.

One simplified, but conceptually accurate, demonstration of the role of the first principles in the derivation of the normal distribution of a psychological characteristic may be given by way of a discussion by Galton (1889) in *Natural Inheritance*. In this discussion, Galton presented, in essence, an example of the random, or drunkard's, walk (Reif, 1965). Galton described an apparatus, a certain quantity of shot, the manner of operation of the apparatus on the shot, and the result obtained as follows:

> [Consider] a frame glazed in front, leaving a depth of about a quarter of an inch behind the glass. Strips are placed in the upper part to act as a funnel. Below the outlet of the funnel stand a succession of rows of pins stuck squarely into the backboard, and below these again are a series of vertical compartments. A charge of small shot is enclosed. When the frame is held topsy-turvy, all the shot runs to the upper end; then, when it is turned back into its working position, the desired action commences. Lateral strips . . . have the effect of directing all the shot that had collected at the upper end of the frame to run into the wide mouth of the funnel. The shot passes through the funnel and issuing from its narrow end, scampers deviously down through the pins in a curious and interesting way; each of them darting a step to the right or left, as the case may be, every time it strikes a pin. The pins are disposed in a quincunx fashion, so that every descending shot strikes against a pin in each successive row. The cascade issuing from the funnel broadens as it descends, and, at length, every shot finds itself caught in a compartment immediately after freeing itself from the last row of pins. The outline of the columns of shot that

> accumulate in the successive compartments approximates to the Curve of Frequency [the normal curve] (pp. 63-64).

Galton (1889) then explained the principle underlying the result obtained with the apparatus and shot:

> The principle on which the action of the apparatus depends is, that a number of small and independent accidents befall each shot in its career. In rare cases, a long run of luck continues to favour the course of a particular shot towards either outside place, but in the large majority of instances the number of accidents that cause Deviation to the right, balance in a greater or less degree those that cause Deviation to the left. Therefore most of the shot finds its way into the compartments that are situated near to a perpendicular line drawn from the outlet of the funnel, and the Frequency with which shots stray to different distances to the right or left of that line diminishes in a much faster ratio than those distances increase. This illustrates and explains the reason why mediocrity is so common (pp. 64-65).

The result of the operation of Galton's apparatus is the Gaussian distribution of the shot with regard to its net displacement. By mediocrity, Galton meant the tendency of the distribution of some characteristic to cluster around the median value of the distribution. In an extension of Galton's argument, the approximately normal distribution of human intelligence, or other psychological characteristic presumed to be so distributed, is due to successive independent random events affecting the individuals making up the population of concern with regard to the formation of their respective intellects or the characteristic of concern.

One might consider the closed system in Galton's example as the shot and apparatus. Any change to either component of the system during its operation would result in the Gaussian distribution of the shot with regard to its net displacement no longer being assured. In the extension of this idea to a psychological characteristic, people take the place of Galton's shot and the apparatus is replaced by the particular series of assumed independent random events affecting the development of this characteristic for each individual. Further, the representative ensemble of systems in Galton's example may without differential effect either be composed of like systems in which each system represents the results of the successive series of independent random events undergone for each particular shot or the results of one particular step in the successive series of independent random events for each shot in the population of concern. Again, in the extension of Galton's example to a psychological characteristic presumed to be approximately normally distributed, people replace his shot and the representative ensemble may then be applied to human systems.

The applicability of the premise that a system composed of many people is closed, that is, a system in which fundamental characteristics of the system remain constant over time, is difficult to entertain seriously for most human systems. It is difficult to conceive a human system, large or small, in the world today that is not unavoidably affected in a significant manner by other external human systems. Second, in those societies which have appeared not to be significantly affected from without, it is certainly questionable whether characteristics of these human systems do not change over time (Jones, 1978; Rowe, 1955). There is evidence, for example, that approximately 3500 years ago, the Tasmanians, island dwelling people who apparently had a closed society for about 12,000 years until the European exploration of their island, stopped eating bony and cartiliginous fish. Jones (1978) was unable to uncover any reasonable ecological basis for their discontinuing their consumption of fish, a readily available food supply. Jones concluded:

> Having disposed of all the plausible ecological explanations that I can think of, I am forced back to a cultural one. I feel that the only possible explanation is that the Tasmanians made an intellectual decision [to stop eating bony and cartiliginous fish] which had the result of constricting their ecological universe (p. 44).

One might also consider a human system composed of all presently existing human systems, this composite human system thus constituting a closed system. There is little doubt that such a composite system would show significant internal change over time. Such a system would differ from the initially analogous physical circumstance in which interacting physical systems that together constitute an isolated physical system and are composed of many particles achieve equilibrium. This equilibrium, reflected in the equal temperatures and chemical potentials of the interacting systems, is considered from a practical standpoint never to change significantly as long as the composite physical system remains isolated. It is this tendency toward equilibrium that is the second law of thermodynamics.

Further, if people have the capacity to order or to find order in experience, any assumption analogous to the fundamental assumption in physics would not be applicable to psychological phenomena. I submit that randomness cannot be the basis for human experience as this experience is inherently ordered at least to some degree. It would certainly not be possible for people to survive without order in their lives. If an assumption analogous to the fundamental assumption were to hold, it is difficult to imagine what the scientific study of psychological phenomena might mean, if instead of the ordered basis that the psychologist adopts for the development of psychological knowledge, randomness was essentially the basis for developing such knowledge. The measurement of intelligence is considered one of the major accomplishments of modern psychology and the tasks found in the various intelligence tests are clearly concerned with the capability of ordering experience. In general, it is fair to say that the more an individual demonstrates the capacity to order experience on these tests, the higher scores will be. Research using various intelligence tests has indicated that intelligence, as measured by these tests, is approximately normally distributed. It is difficult to ascertain what intelligence actually means if an assumption analogous to the fundamental assumption of statistical mechanics and thermal physics, and its attendant randomness, are at the heart of the nature of intelligence. In this regard, the third premise concerning a representative ensemble of systems in statistical mechanics and thermal physics itself constitutes a very sophisticated ordering of experience with an imaginary component that may be correctly applied in the calculation of average values of observable physical quantities. The correctness of this premise in physics appears to refute the applicability of an assumption analogous to the fundamental assumption to human systems in which cognitive phenomena are involved.

In conclusion, in discussing randomness and the normal curve, Galton (1889) wrote:

> It [the law of frequency of errorl reigns with serenity and in complete self-effacement amidst the wildest confusion. The huger the mob, and the greater the apparent anarchy, the more perfect is its sway. It is the supreme law of Unreason. Whenever a large sample of chaotic elements are taken in hand and marshalled in the order of their magnitude, an unsuspected and beautiful form of regularity [the normal curve] proves to have been latent all along (p. 66).

On the Theoretical

Although research has indicated the distributions of certain psychological phenomena such as intelligence as assessed on various intelligence tests are approximately normal, insufficient attention has been paid as a rule, to the importance of theoretically deriving the normal distributions for psychological phenomena from first principles. If one accomplishes this task in a manner similar to that found in statistical mechanics and thermal physics (and I do not know of any other plausible theoretical derivation), the premise is made that randomness lies at the foundation of the psychological phenomena so described. These phenomena are in their very nature indisputably characterized by some degree of order.

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