In this paper, we studied the effects of heat transfer and magnetic field with peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under the assumptions of long wavelength and low Reynolds number. The closed form solutions of velocity field and temperature are obtained. The influence of various pertinent parameters on the flow characteristics, the temperature and the heat transfer coefficient are discussed through graphs.

Such analysis is of great value in medical research. Hayat et al. [14] investigated the peristaltic transport of electrically conducting Maxwell fluid through a porous medium in a planar channel.

In all the above mentioned studies, the interaction of peristalsis with heat transfer has not been taken into account. However, some researchers [15-18] have analyzed the interaction of peristalsis with heat transfer. Very recently, Vasudev et al [19] have investigated the effect of heat transfer on the peristaltic flow of a Newtonian fluid through a porous medium in an asymmetric vertical channel. Recently, Hayat et al [14] have studied the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. So far, the effects of heat transfer and magnetic field on the peristaltic flow of a fluid through a porous medium in a vertical tube have not been studied.

In the present paper, we investigated the effects of heat transfer and magnetic field with peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under the assumptions of long wavelength and low Reynolds number. The closed form solutions of velocity field and temperature are obtained. The influence of various pertinent parameters on the flow characteristics, the temperature and the heat transfer coefficient are discussed through graphs.
2. Mathematical Formulation

We consider the peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube. The flow is generated by sinusoidal wave trains propagating with constant speed $\gamma$ along the wall of the outer tube. A uniform magnetic field $B_0$ is applied in the transverse direction to the flow.

The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to Joule dissipation is neglected. The axisymmetric cylindrical polar coordinate system $(Z,R)$, is chosen such that the $Z$-coordinate is along the center line of the tube and $R$-coordinate along the radial coordinate. The wall of the tube is maintained at a temperature $T$ and at the center we have used axisymmetric condition on temperature. Fig. 1 depicts the physical model of the problem.

The geometry of the tube wall is defined by

$$R = H(Z,t) = a + b \sin \frac{2\pi}{l} [Z - ct]$$  

where $a$ is the radius of the tube, $b$ is the amplitude of the wave, $l$ is the wavelength and $t$ is the time. The flow is unsteady in the fixed frame $(Z,R)$. However, in a coordinate system moving with the propagation velocity $c$ (wave frame $(\tilde{Z}, \tilde{R})$), the boundary shape is stationary. The transformation from fixed frame to wave frame is given by $\tilde{Z} = Z - ct$, $\tilde{R} = R$, $\tilde{W} = W - c\tilde{U} = \tilde{U}$. (2.2)

where $(W, U)$ and $(\tilde{W}, \tilde{U})$ are the velocity components in the wave and fixed frames respectively.

The equations governing the flow in the wave frame of reference are

$$\frac{\partial U}{\partial \tilde{r}} + \frac{U}{\tilde{r}} + \frac{\partial \tilde{W}}{\partial \tilde{z}} = 0$$  

$$\rho \left( \frac{\partial U}{\partial \tilde{r}} + \tilde{W} \frac{\partial U}{\partial \tilde{z}} \right) = \frac{\partial p}{\partial \tilde{r}} + \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial U}{\partial \tilde{r}} \right) + \frac{\partial^2 U}{\partial \tilde{z}^2} \frac{\mu}{k_0}$$  

$$\rho \left( \frac{\partial \tilde{W}}{\partial \tilde{r}} + \frac{\partial U}{\partial \tilde{r}} \right) = -\frac{1}{\tilde{r}} \frac{\partial p}{\partial \tilde{r}} + \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{W}}{\partial \tilde{r}} \right) + \frac{\partial^2 \tilde{W}}{\partial \tilde{z}^2} \left( \frac{\mu}{k_0} + \sigma B_0^2 \right) \left( |w + c| + \rho \sigma \alpha (T - T_0) \right)$$

$$\rho c_p \left( \frac{\partial \tilde{T}}{\partial \tilde{r}} + \frac{\partial \tilde{T}}{\partial \tilde{z}} \right) = k \left( \frac{\partial^2 \tilde{T}}{\partial \tilde{r}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} \right) + Q_0$$

where $p$ is the pressure, $k_0$ is the permeability of the porous medium, $\sigma$ is the electrical conductivity of the fluid, $B_0$ is the magnetic field strength, $T$ is the temperature, $Q_0$ is the constant heat addition/absorption, $\alpha$ is the coefficient of linear thermal expansion of the fluid, $c_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity and $\rho$ is the density of the fluid.

Introducing the non-dimensional variables defined by $\tilde{r} = \frac{r}{a}$, $\tilde{z} = \frac{z}{\tilde{a}}$, $\tilde{W} = \frac{w}{w_0}$, $\tilde{U} = \frac{U}{U_0}$, $\tilde{p} = \frac{p a^2}{\mu \tilde{c}}$, $\tilde{T} = \frac{T - T_0}{T_0}$, $\tilde{\delta} = \frac{\delta}{a}$, $\tilde{h} = \frac{H(z)}{a} = 1 + \phi \sin(2\pi \tilde{z})$, $\phi = \frac{b}{a}$, $Da = \frac{k_0}{a}$, $Re = \frac{\rho a c}{\mu}$, $Gr = gao^2 T_0^2 \tilde{b}$, $Pr = \frac{\mu c_p}{k}$, $\beta = \frac{\alpha Q_0}{kT_0}$, (2.7)

where $Da$ is the Darcy number, $Re$ is the Reynolds number, $\tilde{\delta}$ is the wave number, $\phi$ is the amplitude ratio, $Pr$ is the Prandtl number, $Gr$ is the Grashof number and $\beta$ is the non-dimensional heat source/sink parameter, into the Equations (2.3) – (2.6), we obtain (after dropping the bars)

$$\frac{\partial \tilde{U}}{\partial \tilde{t}} + \tilde{U} \frac{\partial \tilde{U}}{\partial \tilde{r}} + \frac{\partial \tilde{W}}{\partial \tilde{z}} = 0$$

$$\text{Re} \frac{\partial \tilde{W}}{\partial \tilde{r}} = \frac{\partial \tilde{U}}{\partial \tilde{t}} + \frac{\partial \tilde{W}}{\partial \tilde{z}} + \frac{\partial^2 \tilde{U}}{\partial \tilde{z}^2}$$

$$\text{Re} \frac{\partial \tilde{T}}{\partial \tilde{r}} = \frac{\partial \tilde{U}}{\partial \tilde{t}} + \frac{\partial \tilde{T}}{\partial \tilde{z}} + \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} + \tilde{\beta}$$

Using the long wavelength approximation ($\tilde{\delta} \ll 1$) and low Reynolds number ($Re \rightarrow 0$), assumption the Equations (2.9) and (2.11) become

$$\frac{\partial \tilde{p}}{\partial \tilde{z}} = 0$$  

$$\frac{\partial \tilde{p}}{\partial \tilde{z}} = \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{w}}{\partial \tilde{r}} \right) - N^2 \left( \tilde{w} + 1 \right) + Gr \tilde{\theta}$$

$$0 = \frac{\partial \tilde{\theta}}{\partial \tilde{r}} + \frac{1}{\tilde{r}} \frac{\partial \tilde{\theta}}{\partial \tilde{t}} + \tilde{\beta}$$

From Eq. (2.12) and (2.13), we have

$$\frac{\partial \tilde{p}}{\partial \tilde{r}} = \frac{1}{\tilde{r}} \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{w}}{\partial \tilde{r}} \right) - N^2 \left( \tilde{w} + 1 \right) + Gr \tilde{\theta}$$
\[ 0 = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \beta \]  
(2.14)

From Eq. (2.12) and (2.13), we have
\[ \frac{dp}{dz} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) - N^2 (w + 1) + Gr \theta \]  
(2.15)

The corresponding non-dimensional boundary conditions are
\[ \frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 \]  
(2.16)
\[ w = -1 \quad \text{at} \quad r = h \]  
(2.17)
\[ \frac{\partial \theta}{\partial r} = 0 \quad \text{at} \quad r = 0 \]  
(2.18)
\[ \theta = 0 \quad \text{at} \quad r = h \]  
(2.19)

The dimensionless volume flow rate in the wave frame is given by
\[ q = 2 \int_0^1 w r dr \]  
(2.20)

The dimensionless instantaneous volume flow rate in the fixed frame of reference is given by
\[ Q(x,t) = 2 \int_0^1 w r dr = 2 \int_0^1 (w + 1) r dr = q + h^2 \]  
(2.21)

The dimensionless time mean flow over a period \[ T = \frac{\lambda}{c} \] of the peristaltic wave, is defined as
\[ \overline{Q} = \frac{1}{T} \int Q(x,t) dt = q + \frac{1}{2} h^2 dx = q + 1 + \frac{\theta^2}{2} \]  
(2.22)

3. SOLUTION

Solving Eq. (2.14) using the boundary conditions (2.18) and (2.19), we get
\[ \theta = \frac{\beta}{4} \left( h^2 - r^2 \right) \]  
(3.1)

Substituting Eq. (3.1) in to the Eq. (2.15) and solving Eq.(2.15) together with the boundary conditions (2.16) and (2.17), we obtain
\[ w = \frac{1}{N^2} \left( l_N(Nh) - 1 \right) - \frac{1}{4N^2} \left( h^2 - r^2 \right) + \frac{4}{N^2} \left( l_N(Nh) - 1 \right) \]  
(3.2)

The volume flow rate \[ q \] is given by
\[ q = \frac{h}{N^2} \int_0^1 \left[ 2I_1(Nh) - hNl_N(Nh) \right] - h^2 + Gr \beta \frac{A_4}{8N^2} \]  
(3.3)

where \[ A_4 = h^2 + r^2 + \frac{8h}{N^2} \int_0^1 \left[ 2I_1(Nh) - hNl_N(Nh) \right] \]

From Eq.(3.3), we have
\[ \frac{dp}{dz} = \frac{N^2}{h^2} \left[ 2I_1(Nh) - hNl_N(Nh) \right] \]  
(3.4)

The pressure rise \[ \Delta p \] per one wave length is given by
\[ \Delta p = \int_0^1 \frac{dp}{dz} dz \]  
(3.5)

The heat transfer coefficient at the outer wall is defined by
\[ \zeta = \frac{\partial \theta}{\partial r} \bigg|_{r=h} = -a \phi h \pi \cos(2 \pi z) \]  
(3.6)

4. Discussion of The Results

In order to see the quantitative effects of the various emerging parameters involved in the results on the pumping characteristics and the heat transfer coefficient we use the MATLAB package.

Fig. 2 shows the variation of pressure rise \[ \Delta p \] with the time - averaged flux \[ \overline{Q} \] for different values of \[ \beta \] with \[ \phi = 0.6, Gr = 3, Da = 0.1 \text{ and } M = 1 \]. It is found that, the time-averaged flux \[ \overline{Q} \] increases with increasing heat source/sink parameter \[ \beta \] in all the three (pumping \((\Delta p > 0)\), free pumping \((\Delta p = 0)\) and co-pumping \((\Delta p < 0)\) ) regions.

The variation of pressure rise \[ \Delta p \] with the time - averaged flux \[ \overline{Q} \] for different values of \[ Gr \] with \[ \phi = 0.6, \beta = 5, Da = 0.1 \text{ and } M = 1 \] is depicted in Fig. 3. It is found that, an increase in the \[ Gr \] increases the time - averaged flux \[ \overline{Q} \] in all the three (pumping free-pumping and co-pumping) regions.

Fig. 4 presents the variation of pressure rise \[ \Delta p \] with the time - averaged flux \[ \overline{Q} \] for different values of \[ Da \] with \[ \phi = 0.6, Gr = 3, \beta = 3 \text{ and } M = 1 \]. It is noticed that, the time-averaged flux \[ \overline{Q} \] decreases with increasing \[ Da \] in the pumping region, while \[ \overline{Q} \] increases with increasing \[ Da \] in the free pumping and co-pumping regions.

The variation of pressure rise \[ \Delta p \] with the time - averaged flux \[ \overline{Q} \] for different values of \[ M \] with \[ \phi = 0.6, Gr = 3, Da = 0.1 \text{ and } \beta = 5 \text{ is shown in Fig. 5. It is observed that, any two pumping curves intersect in first quadrant to the left of this point of intersection the \[ \overline{Q} \] increases with increasing \[ M \].}

5. CONCLUSIONS

In this chapter, we modeled the effects of magnetic field and heat transfer on the peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under long wavelength and low Reynolds number approximations. The closed form solutions of velocity field and temperature are obtained.

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5. Conclusions

In this chapter, we modeled the effects of magnetic field and heat transfer on the peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under long wavelength and low Reynolds number approximations. The closed form solutions of velocity field and temperature are obtained. It is found that, in the pumping region, the time-averaged volume flux increases with increasing $\beta$, $Gr$, $M$ and $f$, while it decreases with an increase in $Da$. The temperature $\theta$ increases with increasing $\beta$ and $f$. Also, the heat transfer coefficient $Z$ increases with increasing $\beta$ and $f$.

Fig. 6 depicts the variation of pressure rise $\Delta p$ with the time-averaged flux $\bar{Q}$ for different values of $\phi$ with $\beta = 5$, $Gr = 3$, $Da = 0.1$ and $M = 1$. It is found that, the $\bar{Q}$ increases with increasing $f$, in both pumping and free pumping regions, while in co-pumping region, the $\bar{Q}$ decreases as $\phi$ increases, for an appropriately chosen $\Delta p (< 0)$.

Fig. 7 shows the temperature profiles for different values of $\beta$ with $\phi = 0.6$ and $z = 0.1$. It is observed that, the temperature $\theta$ increases with increasing $\beta$.

Temperature profiles for different values of $\phi$ with $\beta = 5$ and $z = 0.1$ is presented in Fig. 8. It is found that, the temperature $\theta$ increases with an increase in $\phi$. In order to see the effects of $\beta$ and $\phi$ on the heat transfer coefficient $Z$ at the tube wall we have computed numerically and are presented in Table 1-2. Table 1 presents that, the heat transfer coefficient $Z$ increases with increasing $\beta$. From Table 2, we observed that the heat transfer coefficient $Z$ increases with increasing $\phi$.

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6. References


