Int J Cur Sci Res. 2011; 1(3): 105- 110

Contents lists available at CurrentSciDirect Publications

International Journal of Current Scientific Research

Journal homepage: www.currentscidirect.com

Original article

Peristaltic flow of a Newtonian fluid through a porous medium in a vertical tube under the effect of a magnetic field

***C.Vasudev[a] , Prof.U.Rajeswara Rao[b] , G.Prabhakara Rao[c] , M.V.Subba Reddy[d]**

[a] *Research Scholar, Department of Mathematics, Sri Krishna Devaraya University, Anantapur, India.*
[a] *Assistant Professor, Department of Mathematics, K.S.Institute of Technology, Bangalore, India.*
[b] *Head, Chairman & BOS, Department of Mathematics, Sri Krishna Devaraya University, Anantapur, India.*
[c] *Research Scholar, Department of Mathematics,Sri Krishna Devaraya University, Anantapur, India.*
[d] *Professor, Department of Information Technology,Sri Venkatesa Perumal College of Engineering, Puttur,*

ARTICLE INFO

Keywords:
Heat Transfer
Magnetic Field
Newtonian Fluid
Porous Medium
Reynolds Number

ABSTRACT

In this paper, we studied the effects of heat transfer and magnetic field with peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under the assumptions of long wavelength and low Reynolds number. The closed form solutions of velocity field and temperature are obtained. The influence of various pertinent parameters on the flow characteristics, the temperature and the heat transfer coefficient are discussed through graphs.

1. Introduction

The peristaltic transport through tubes /channels have attracted considerable attention due to their wide applications in medical and engineering sciences, such as, in physiology, roller and finger pumps, sanitary fluid transport, transport of corrosive fluids etc. A number of analytical [1-6], as well as numerical and experimental [7-11] studies of peristaltic flows of different fluids have been reported. Several review articles have been reported by [5].

Flow through a porous medium have several practical applications especially in geophysical fluid dynamics. Examples of natural porous media are beach sand, sandstone, limestone, the human lung, bile duct, gall bladder with stones in small blood vessels. El Shehawey and Husseny [12] and El Shehawey et al. [13] studied the peristaltic flow of a Newtonian fluid thorough a porous medium. Furthermore, the MHD effect on peristaltic flow is important in technology (MHD pumps) and biology (blood flow).

Such analysis is of great value in medical research. Hayat et al. [14] investigated the peristaltic transport of electrically conducting Maxwell fluid through a porous medium in a planar channel.

In all the above mentioned studies, the interaction of peristalsis with heat transfer has not been taken into account. However, some researchers [15-18] have analyzed the interaction of peristalsis with heat transfer. Very recently, Vasudev et al [19] have investigated the effect of heat transfer on the peristaltic flow of a Newtonian fluid through a porous medium in an asymmetric vertical channel. Recently, Hayat et al [14] have studied the effect of heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. So far, the effects of heat transfer and magnetic field on the peristaltic flow of a fluid through a porous medium in a vertical tube have not been studied.

In the present paper, we investigated the effects of heat transfer and magnetic field with peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under the assumptions of long wavelength and low Reynolds number. The closed form solutions of velocity field and temperature are obtained. The influence of various pertinent parameters on the flow characteristics, the temperature and the heat transfer coefficient are discussed through graphs.

* Corresponding Author : C.Vasudev
Assistant Professor,
Department of Mathematics,
K.S. Institute of Technology, Bangalore, India.
E-mail:raadha222@gmail.com

2. Mathematical Formulation

We consider the peristaltic flow of a viscous incompressible Newtonian fluid through a porous a medium in a vertical tube. The flow is generated by sinusoidal wave trains propagating with constant speed c along the wall of the outer rube. A uniform magnetic field B0 is applied in the transverse direction to the flow.

The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to Joule dissipation is neglected. The axisymmetric cylindrical polar coordinate system (*ZR*), is chosen such that the *Z*-coordinate is along the center line of the tube and *R*- coordinate along the radial coordinate. The wall of the tube is maintained at a temperature to and at the center we have used axisymmetric condition on temperature. Fig. 1 depicts the physical model of the problem.

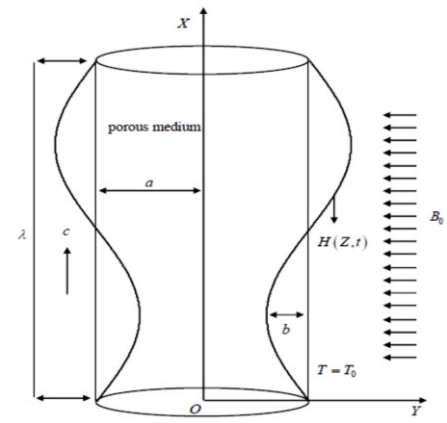

Fig. 1 The Physical Model

The geometry of the tube wall is defined by

$$R = H(Z,t) = a + b\sin\frac{2p}{l}\left[Z - ct\right] \qquad (2.1)$$

where a is the radius of the tube, b is the amplitude of the wave, λ is the wavelength and t is the time.

The flow is unsteady in the fixed frame (Z, R). However, in a co-ordinate system moving with the propagation velocity c (wave frame (z, r)), the boundary shape is stationary. The transformation from fixed frame to wave frame

is given by $z = Z - ct,\ r = R,\ w = W - c,\ u = U$ (2.2)

where (w, u) and (W, U) are the velocity components in the wave and fixed frames respectively.

The equations governing the flow in the wave frame of reference are

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad (2.3)$$

$$\rho\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} - \frac{\mu}{k_0}u \qquad (2.4)$$

$$\rho\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2} - \left[\frac{\mu}{k_0} + \sigma B_0^2\right](w + c) + \rho g\alpha(T - T_0) \qquad (2.5)$$

$$\rho c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + Q_0 \qquad (2.6)$$

where p is the pressure, k_0 is the permeability of the porous medium, σ is the electrical conductivity of the fluid, B_0 - magnetic field strength, T is the temperature , Q_0 is the constant heat addition/absorption, α is the coefficient of linear thermal expansion of the fluid, c_p is the specific heat at constant pressure, k is the thermal conductivity and ρ is the density of the fluid.

Introducing the non-dimensional variables defined by $\bar{r} = \frac{r}{a}$,

$$\bar{z} = \frac{z}{\lambda},\ \bar{w} = \frac{w}{c},\ \bar{u} = \frac{u}{c\delta},\ \bar{p} = \frac{pa^2}{\mu c\lambda},$$

$$\bar{S} = \frac{aS}{\mu c},\quad \theta = \frac{T - T_0}{T_0},\quad \delta = \frac{a}{\lambda},\ h = \frac{H(z)}{a} = 1 + \phi\sin(2\pi z),\quad \phi = \frac{b}{a},$$

$$Da = \frac{k_0}{a^2},\ \mathrm{Re} = \frac{\rho ac}{\mu},\ Gr = \frac{g\alpha a^3 T_0}{\nu^2},\ \Pr = \frac{\mu c_p}{k},\ \beta = \frac{a^2 Q_0}{kT_0} \qquad (2.7)$$

where Da is the Darcy number, Re is the Reynolds number, δ is the wave number, ϕ is the amplitude ratio, $\Pr$ is the Prandtl number, Gr is the Grashof number and β is the non-dimensional heat source/sink parameter, into the Equations (2.3) – (2.6), we obtain (after dropping the bars)

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad (2.8)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial r} + \frac{\delta}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \delta^2\frac{\partial^2 u}{\partial z^2} - \frac{\delta}{Da}u \qquad (2.9)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \delta\frac{\partial^2 w}{\partial z^2} - N^2(w + 1) + Gr\theta \qquad (2.10)$$

$$\mathrm{Re}\Pr\delta\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \delta^2\frac{\partial^2 T}{\partial z^2} + \beta \qquad (2.11)$$

where $N^2 = \frac{1}{Da} + M^2$.

Using the long wavelength approximation $(\delta << 1)$ and low Reynolds number $(\mathrm{Re} \to 0)$, assumption the Equations (2.9) and (2.11) become

$$\frac{\partial p}{\partial r} = 0 \qquad (2.12)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - N^2(w + 1) + Gr\theta \qquad (2.13)$$

$$0 = \frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \beta \qquad (2.14)$$

From Eq. (2.12) and (2.13), we have

$$\frac{dp}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - N^2(w + 1) + Gr\theta \qquad (2.15)$$

$$0=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\beta \tag{2.14}$$

From Eq. (2.12) and (2.13), we have

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)-N^2\left(w+1\right)+Gr\theta \tag{2.15}$$

The corresponding non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r}=0 \quad \text{at} \quad r=0 \tag{2.16}$$

$$w=-1 \quad \text{at} \quad r=h \tag{2.17}$$

$$\frac{\partial\theta}{\partial r}=0 \quad \text{at} \quad r=0 \tag{2.18}$$

$$\theta=0 \quad \text{at} \quad r=h \tag{2.19}$$

The dimensionless volume flow rate in the wave frame is given by

$$q=2\int_0^h wrdr \tag{2.20}$$

The dimensionless instantaneous volume flow rate in the fixed frame of reference is given by

$$Q(x,t)=2\int_0^h WRdr=2\int_0^h(w+1)rdr=q+h^2 \tag{2.21}$$

The dimensionless time mean flow over a period $T\left(=\lambda/c\right)$ of the peristaltic wave, is defined as

$$\overline{Q}=\frac{1}{T}\int_0^T Q(x,t)dt=q+\int_0^1 h^2dx=q+1+\frac{\phi^2}{2} \tag{2.22}$$

3. SOLUTION

Solving Eq. (2.14) using the boundary conditions (2.18) and (2.19), we get

$$\theta=\frac{\beta}{4}\left[h^2-r^2\right] \tag{3.1}$$

Substituting Eq. (3.1) in to the Eq. (2.15) and solving Eq.(2.15) together with the boundary conditions (2.16) and (2.17), we obtain

$$w=\frac{1}{N^2}\frac{dp}{dz}\left[\frac{I_0(Nr)}{I_0(Nh)}-1\right]-1+\frac{Gr\beta}{4N^2}\left[\left(h^2-r^2\right)+\frac{4}{N^2}\left(\frac{I_0(Nr)}{I_0(Nh)}-1\right)\right] \tag{3.2}$$

The volume flow rate q is given by

$$q=\frac{h}{N^3I_0(Nh)}\frac{dp}{dz}\left[2I_1(Nh)-hNI_0(Nh)\right]-h^2+\frac{Gr\beta}{8N^2}A_1 \tag{3.3}$$

where $A_1=h^4+\frac{8h}{N^3I_0(Nh)}\left[2I_1(Nh)-hNI_0(Nh)\right]$.

From Eq.(3.3), we have

$$\frac{dp}{dz}=\frac{N^3I_0(Nh)\left(q+h^2-\frac{Gr\beta A_1}{8N^2}\right)}{h\left[2I_1(Nh)-hNI_0(Nh)\right]} \tag{3.4}$$

The pressure rise Δp per one wave length is given by

$$\Delta p=\int_0^1\frac{dp}{dz}dz \tag{3.5}$$

The heat transfer coefficient at the outer wall is defined by

$$\zeta=\frac{\partial\theta}{\partial r}\frac{\partial r_2}{\partial z}\bigg|_{y=h_1}=-\phi\beta h\pi\cos\left(2\pi z\right) \tag{3.6}$$

4. Discussion of The Results

In order to see the quantitative effects of the various emerging parameters involved in the results on the pumping characteristics and the heat transfer coefficient we use the MATLAB package.

Fig. 2 shows the variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of β with $\phi=0.6$, $Gr=3$, $Da=0.1$ and $M=1$.It is found that, the time-averaged flux $\overline{Q}$ increases with increasing heat source/sink parameter β in all the three (pumping $\left(\Delta p>0\right)$, free pumping $\left(\Delta p=0\right)$ and co-pumping $\left(\Delta p<0\right)$) regions.

The variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of Gr with $\phi=0.6$, $\beta=5$, $Da=0.1$ and $M=1$ is depicted in Fig. 3. It is found that, an increase in the Gr increases the time - averaged flux $\overline{Q}$ in all the three (pumping, free-pumping and co-pumping) regions.

Fig. 4 presents the variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of Da with $\phi=0.6$, $Gr=3$, $\beta=5$ and $M=1$. It is noticed that, the time-averaged flux $\overline{Q}$ decreases with increasing Da in the pumping region, while $\overline{Q}$ increases with increasing Da in the free pumping and co-pumping regions.

The variation of pressure rise Δp with the time – averaged flux $\overline{Q}$ for different values of M with $\phi=0.6$, $Gr=3$, $Da=0.1$ and $\beta=5$ is shown in Fig. 5. It is observed that, any two pumping curves intersect in first quadrant to the left of this point of intersection the $\overline{Q}$ increases with

5. CONCLUSIONS

In this chapter, we modeled the effects of magnetic field and heat transfer on the peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under long wavelength and low Reynolds number approximations. The closed form solutions of velocity field and temperature

ACKNOWLEDGEMENTS

We thank the management of Sri Kalki Supreme Constructions Pvt Ltd Hyderabad; Suma Engineering works Bangalore, Sri Sanjana Food Products Chittoor, Swami & sons constructions

increasing M while to the right side of this point of intersection the $\overline{Q}$ decreases with increasing M .

Fig. 6 depicts the variation of pressure rise Δp with the time – averaged flux $\overline{Q}$ for different values of ϕ with $\beta = 5, Gr = 3,\ Da = 0.1$ and

$M = 1$. It is found that, the $\overline{Q}$ increases with increasing in f , in both pumping and free pumping regions, while in co-pumping region,

the $\overline{Q}$ decreases as ϕ increases, for an appropriately chosen $\Delta p(<0)$.

Fig.7 shows the temperature profiles for different values of β with $\phi = 0.6$ and $z = 0.1$.

It is observed that, the temperature θ increases with increasing β.

Temperature profiles for different values of ϕ with $\beta = 5$ and $z = 0.1$ is presented in Fig. 8.

It is found that, the temperature θ increases with an increase in ϕ . In order to see the effects

of β and ϕ on the heat transfer coefficient Z at the tube wall we have compute numerically and are presented in Table 1- 2. Table 1 presents that, the heat transfer coefficient Z increases with increasing β. From Table 2, we observed that the heat transfer coefficient Z increases with increasing ϕ .

are obtained. It is found that, in the pumping region, the time-averaged volume flux increases with increasing β, Gr , M and ϕ , while it decreases with an increase in Da . The temperature θ increases with increasing β and ϕ . Also, The heat transfer coefficient Z increases with increasing β and ϕ .

Bangalore, Sri Sai Educational Institutions Anantapur, India, for their support, valuable guidelines, consistent encouragement and providing me necessary facilities in pursuing the research-work.

5. Conclusions

In this chapter, we modeled the effects of magnetic field and heat transfer on the peristaltic flow of a viscous incompressible Newtonian fluid through a porous medium in a vertical tube under long wavelength and low Reynolds number approximations. The closed form solutions of velocity field and temperature are obtained. It is found that, in the pumping region, the time-averaged volume flux increases with increasing β, Gr , M and ϕ, while it decreases with an increase in Da. The temperature θ increases with increasing β and ϕ. Also, The heat transfer coefficient Z increases with increasing β and ϕ.

6. Acknowledgements

We thank the management of Sri Kalki Supreme Constructions Pvt Ltd Hyderabad; Suma Engineering works Bangalore, Sri Sanjana Food Products Chittoor, Swami & sons constructions Bangalore, Sri Sai Educational Institutions Anantapur, India, for their support, valuable guidelines, consistent encouragement and providing me necessary facilities in pursuing the research-work.

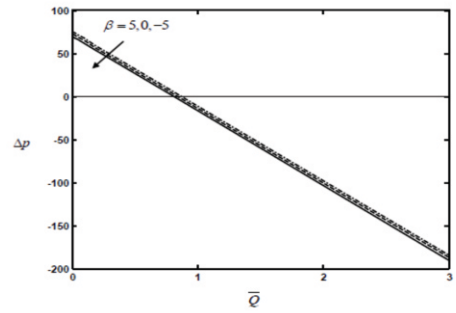

Fig. 2. The variation of pressure rise Δp with the time –averaged flux $\overline{Q}$ for different values of β with $\phi = 0.6, Gr = 3,\ Da = 0.1$ and $M = 1$.

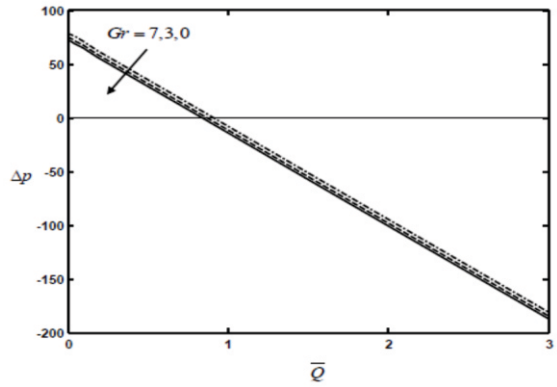

Fig. 3. The variation of pressure rise Δp with the time –averaged flux $\overline{Q}$ for different values of Gr with $\phi = 0.6, \beta = 5,\ Da = 0.1$ and $M = 1$.

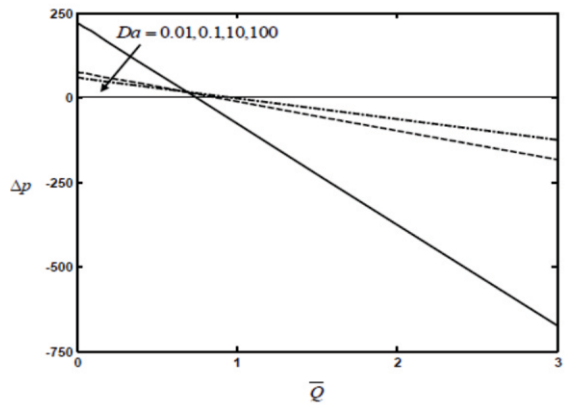

Fig. 4. The variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of Da with $\phi = 0.6,\ Gr = 3,\ \beta = 5$ and $M = 1$.

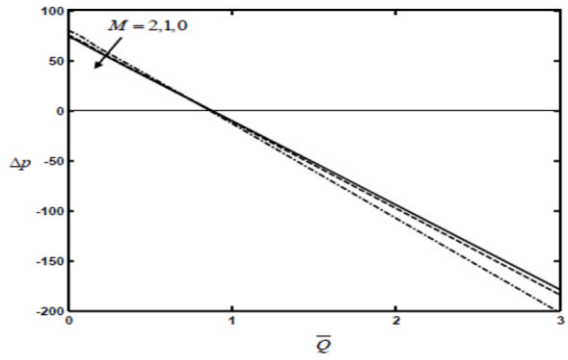

Fig. 5. The variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of M with $\phi = 0.6, Gr = 3,\ Da = 0.1$ and $\beta = 5$.

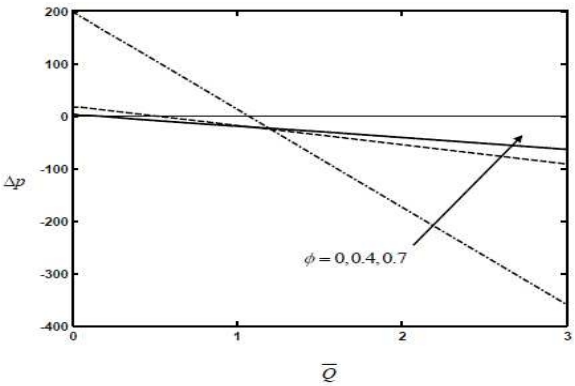

Fig. 6. The variation of pressure rise Δp with the time - averaged flux $\overline{Q}$ for different values of ϕ with $\beta = 5$, $Gr = 3$, $Da = 0.1$ and $M = 1$.

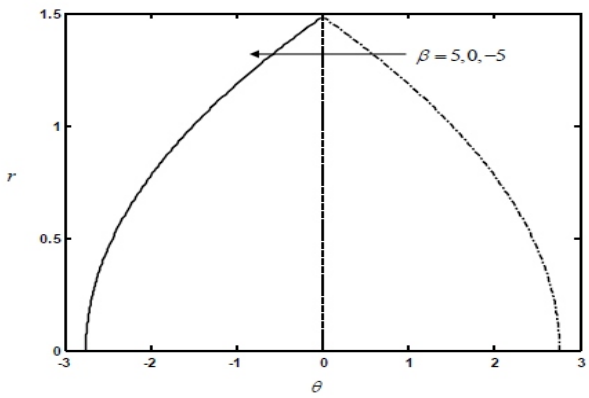

Fig.7. Temperature profiles for different values of β with $\phi = 0.6$ and $z = 0.1$.

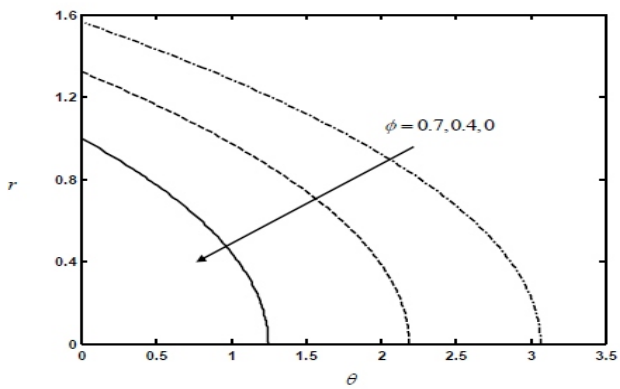

Fig. 8. Temperature profiles for different values of ϕ with $\beta = 5$ and $z = 0.1$.

x	β		
	-5	0	5
0.1	-7.4935	0	7.4935
0.2	-14.0784	0	14.0784
0.3	-14.0784	0	14.0784

Table 1. The variation of heat transfer coefficient Z with β for $\phi = 0.6$.

x	ϕ		
	0	0.4	0.7
0.1	0	4.5615	9.1223
0.2	0	8.2489	17.4193
0.3	0	8.2489	17.4193

Table 2. The variation of heat transfer coefficient Z with ϕ for $\beta = 5$.

6. References

[1] Burns J C, Parkes T. Peristaltic motion. J Fluid Mech. 1967; 29:731-743.
[2] Fung Y C, Yih C S. Peristaltic transport, Trans. ASME J Appl Mech. 1968; 35: 669-675.
[3] Jaffrin M Y. Inertia and streamline curvature effects on peristaltic pumping. Int J Engng Sci.1973; 11: 681-699.
[4] Ramachandra Rao A, Usha S. Peristaltic transport of two immiscible viscous fluid in a circular tube. J Fluid Mech. 1995; 298: 271-285.
[5] Shapiro A H, Jaffrin MY, Weinberg S L. Peristaltic pumping with long wavelengths at low Reynolds number. J Fluid Mech. 1969; 37:799-825.
[6] Siddiqui AM, Schwarz WH. Peristaltic flow of a second order fluid in tubes, J. Non-Newtonian Fluid Mech. 1994; 53:257-284.
[7] Latham TW. Fluid motions in peristaltic pump, MS Thesis, MIT, Cambridge, Massachussetts, 1966.
[8] Weinberg SL, Eckistein EC, Shapiro AH. An experimental study of peristaltic pumping. J Fluid Mech. 1971; 49: 461-497.
[9] Takabatake S, Ayukawa K. Numerical study of two-dimensional peristaltic flow. J Fluid Mech. 1982;122: 439-465.
[10] Takabatake S, Ayukawa K, Mori A. Peristaltic pumping in circular tubes: A numerical study of fluid transport and its efficiency. J Fluid Mech. 1988;193: 267-283.
[11] Tang D, Rankin S. Numerical and asymptotic solutions for peristaltic motion of nonlinear viscous flows with elastic free boundaries. Siam J Sci Comput. 1993; 14 :1300-1319.
[12] El Shehawey EF, Mekheimer Kh S, Kaldas S F, Afifi N A S. Peristaltic transport through a porous medium. J Biomath.1999; 14 .

[13] El Shehawey EF, Sobh AMF, Elbarbary EME. Peristaltic motion of a generalized Newtonian fluid through a porous medium . Phys Soc Jpn. 2000; 69: 401-407.
[14] Hayat T, Ali N, Asghar S. Hall effects on peristaltic flow of a Maxwell fluid in a porous medium. Physics Letters A. 2007;363: 397- 403.
[15] Vajravelu K, Radhakrishnamacharya G, Radhakrishnamurthy V. Peristaltic flow and heat transfer in a vertical porous annulus, with long wave approximation. Int J Non-Linear Mech.2007; 42:754-759.
[16] Radhakrishnamacharya G, Srinisvasulu Ch. Influence of wall properties on peristaltic transport with heat transfer. CR Mecanique. 2007; 335: 369-373.
[17] Mekheimer KHS, Abd elmaboud Y. The influence of heat transfer and magnetic field on peristaltic transport of a Newtonian .uid in a vertical annulus: Application of an endoscope. Phys Letts A. 2008; 372:1657-1665.
[18] Nadeem S, Akbar N S. Effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a non-uniform vertical tube, Commun. Non-linear Sci Numer Simulat. 2010;15: 3950–3964.
[19] Vasudev C, Rajeswara Rao U, Subba Reddy M V, Prabhakara Rao G. Effect of heat transfer on peristaltic transport of a Newtonian fluid through a porous medium in an asymmetric vertical channel. European Journal of Scientific Research. 2010;44: 79 - 92.