Power laws in Elections
A Survey

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Abstract
Empirical power laws in general elections are surveyed in Brazil, Mexico, India, and with focal analysis in Indonesia. The diversity of preference dynamics in voter’s social network and the way multi-party systems to be in its critical conditions are responsible for this pushing around the evolution of political system at general. For the special case of Indonesia, we report the existing robustness for levels of legislative elections throughout 1999 to 2009. We show that the scale free phenomena in different scales of aggregation and outline some analytical outlines and consequences that could be drawn referring to this.

Keywords: democracy, elections, power law.
1. Introduction

Applications of physics in analytical system in general are not a new. It has been started in inspiring and invigorating since the development of sociology to understand social phenomena [1]. Collective behavior, phase transition, critical condition, and power laws are stylized characteristics of complex system identified empirically observable in social phenomena. Complex perspectives to explain the interacting social agents forming the social system have made statistical mechanics challenging to identify and extract complex characteristics of social life.

General Election has been widely recognized a democratic way to periodically evaluate the national living. Through this mechanism, individuals have the right freely expressing their choices on candidates to be seated as presidents, leaders, or legislative representatives. However, it is interesting to find out that even in the most democratic processes; individuals are bounded to the so-called social and political influences upon which they live. Social system “guides” the voters’ preferences and gives task to optimize choices as seen in multi-parties election. Consequently, the dynamics of preferences would emerge the result of election as collective phenomena that are sometimes counter-intuitive to the micro characteristics of agents for the embedded non-linearity in social system.

Complex properties in voting and election are characterized by the existing scale-free phenomena in the rank distribution of the tally. Empirical studies throughout countries confirm that there is inversely power relation between the rank of candidates/parties to the votes gained respectively: $N(v) \sim v^{-\alpha}$ [2-7] where $\alpha$ denotes the scaling parameters varying throughout countries. The similar distributional patterns be it spatially or chronologically indicates a teasing conjectures for the general mechanisms that might lead to the basic form in which micro-social decision makings laid upon.

Surveying the persistence of the power laws in tallies are the objective of the presentation. We begin by travelling throughout empirical works reporting the phenomena across some countries and certainly with different political systems yet involving many political parties. This is followed by analyzing the election for legislative chairs in Indonesia for the last three periods of which has been reported previously in [7]. The paper is ended with discussions related to the power laws and opinion dynamics formation witnessed in the General Election and some concluding remarks focusing on the democratic party in Indonesia.

2. Surveys

II. 1. Brazillian elections.

In Brazil, the phenomena of the power law is primarily reported in [2,3], on the empirical data of legislative election for the working period 1998-2003. The Brazillian legislative election is to choose those to be seated in the Chamber of Federal Deputies held by employing the open-list proportional representation system. In this mechanism, the gained party’s chairs are proportional to the total votes for candidates in each federal state, while the most voted candidates in the party’s list would be seated for the legislative institution of federal state.

By sorting all of the candidates based on the fraction of their votes normalized to the total voters in each federal state, it has been shown that the histogram of the candidates ($N$) has a relation of power law $N(v) \propto v^{-\alpha}$ with $v$, gained votes (Fig 1a). Interestingly, this pattern is also seen in the rank distribution in the election of for federal and state deputies nationally or in a particular state for two periods of election with power law exponent $\alpha \approx 1$ [2,3]. It is mind teasing to see this universality
comes from the similarity of the voters’ background on their decision despite the different mechanisms of both elections.

Using the similar procedures, Lyra et.al [4] reported the scale-free characteristics in the election for city councils in Brazil. However, unlike Filho et.al, histogram data of candidates, $N$, gaining fraction of vote, $v$, in City Council election is fitted with generalized Zipf law, namely $N(v) \propto (1 + Cv)^\alpha$, yielding exponent of $\alpha = 2.6$, a similar result on each federal state. This shows that election process in these localities is sensitive to the social characteristics on each federal state. In the other hand, the power law exponents in the levels of City Council are much bigger relative to those in federal or states, an indication of different voting dynamics in both.

II. 2. Mexican Elections

Legislative election in Mexico is held every three years pointing 500 representatives for Chamber of Deputies. Here, 300 legislatures are chosen in majority votes based on single member district mechanism, while the rest is pointed from proportional representation of national scale votes for political parties.

Morales et.al [5] analyzed legislative elections in Mexico for 5 cycles of elections (1991-2003) and reported the scale free in the tally for both mechanisms. For each distribution, fraction of votes for candidates and parties are normalized to the total votes of both. Cumulative distribution of candidates and parties gaining votes of $v$ ($P(V > v) \sim v^{-\theta}$) exhibits power-law behavior with different exponents $\alpha = \theta + 1$ ($\alpha_{\text{neighbors}} = 1.33$, $\alpha_{\text{candidates}} = 1.23$, $\alpha_{\text{parties}} = 1.4475$). Moreover, they conclude that Mexican voters dynamics can be characterized into three distribution exponent for its universality in 5 cycles of election, nationally as well as partially in each federal state. Thus, as it has been discussed in Brazilian election, universality discovered in Mexican one indicates that the macro-view of how voters put their votes is not really affected by the difference of social or geographic condition upon which the voters are.
II. 3. Indian Elections
The legislative election in India is held by the fundamentals of the mechanism, called the single member district system. Here, all districts conduct the general elections to choose their representatives in the House. Overall, there are 543 legislatures in the Indian House of Representatives (Lok Sabha), each represents an electoral district and chosen in the mechanism of plurality voting.

Gonzales, et. al. reports that the histogram of the total candidates with the fraction of votes, \( v \), in 1998 Indian election follows the power law [6]. This phenomenon is found at all aggregation levels, be it national, partial in each federal state, and district. In each aggregative level, histogram is reconstructed in normalizing the gained votes of each candidate to the total votes in the respective level. Interestingly, the results are quite different with those had found in Brazil, the exponents are different in each aggregative level and in federal state as well (\( \alpha_{\text{national}}=1.3, \alpha_{\text{Uttar Pradesh}}=1.32, \alpha_{\text{Goa}}=0.97, \alpha_{\text{andhra Pradesh}}=1.51, \alpha_{\text{Haryana}}=2.06, \alpha_{\text{Kerala}}=1.26, \alpha_{\text{election district}}=1 \)) (fig. 3).

II. 4. Indonesia
The mechanism of Indonesian election for legislatures has fundamentally changed in the last three cycles of general elections. In 1999, the legislative election for the House of Representatives (Dewan Perwakilan Rakyat, DPR) be it national-scale representation, or provincial regions and municipalities are held with closed list proportional representation by using the generalized district system. This mechanism chooses the candidates by referring to the sequence number in the party list. This has been changed in General Election 2004 that incorporated the open list proportional representative,
electing the candidates whose gain a threshold number of votes. A change is implemented again since 2004 by incorporating the representational legislative system of bicameralism (people are also represented by non-party senators by majority votes).

![Figure 3](image)

**Figure 3**

The scale free is shown in Indonesian election. This is shown by normalizing the votes gained by parties and candidates in each province to the respective majorities and then combine them as histogram of the gained votes fraction \( N(v) \) nationally [7]. The power law is shown to have exponent \( \alpha_{\text{DPR1999}} = 1.632, \alpha_{\text{DPR 2004}} = 1.41 \) dan \( \alpha_{\text{DPD 2004}} = 1 \) (fig 4 & 5). An expansion to the one presented in [7] is shown in the paper. We show the statistical pattern in Indonesian legislative election in 1999, 2004, and 2009, due to national as well as provincial respect.

![Figure 4](image)

**Figure 4**

In the 2009 election, the power law pattern is also exhibited with exponents not far from the reported previous election, \( \alpha_{\text{DPR2009}} = 1.46 \) (fig 6a (i)). It is interesting to note that the same patterns are also shown in election district (fig. 6a (ii)) and municipals (fig. 6a (iii)). For the figures, it is
apparent that the smoother patterns are shown for lower aggregative levels – things that are also observable in the last two elections.

With the similar observation, power laws could also be seen in three elections for provincial regions (fig. 6b,7b,8b). In some cases, similar findings with the one in India related to the varied exponents in regions of province. One could relate this to the semi-district election or the archipelagic geographical situation in Indonesia.

In advance, the votes for parties in regional elections normalized to the sub-municipal are also exhibited the pattern (fig. 9). In one hand, the emerging power law shows the typicality of complex system on which the elections are held, while in the other hand different exponents reflects heterogeneous micro-structure on each of the election.

In Indonesia, while the power law is robust from elections to elections, the exponent of elections in 2004 and 2009 is very different (larger than) with the finding in 1999. This indicates there are different social structure on which the three elections were held. Fundamentally speaking, this is
plausible for the 1999 election is a sort of transition to the democratic general election held since 1955. With more detail exploration to the result of 1999 election, the result also shows two different power law exponents, $\alpha_{\text{head}} = 1.68235$ dan $\alpha_{\text{tail}}=1.19936$, at the same votes rank. Such two regimes of power law are also found in words rank distribution as in British National Corpus [12]. Two regimes of power law exponents characterizes the competition among 48 political parties, of which most of them are a new in Indonesian political stage. First regime is made by domination of several majority parties while the other reflects the tight competition among new political parties with exponent $\alpha=1$.

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Gambar 9
Power laws in Indonesian Provincial House of Representative 2004 in (i). Province Aceh, $\alpha=1.24$, (ii). Province Banten, $\alpha=1.16$, (iii). Central Java, $\alpha=1.24$ (iv). East Java, $\alpha=1.29$

Gambar 10:
Two regimes of power laws in Indonesian House of Representative election 1999

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3. Discussions

By using democratic processes as a language, we can say that election result could be a kind of sentence representing the consensus of the social system due to political living – here, the political parties and candidates are the words to be chosen to aspire what the people want. By ranking down the political parties and candidates due to the candidates/party \( n \) and the gained votes \( V \), we have a relation of power law,

\[
V(n) \propto n^{-\beta}
\]  

(1)

which is the well-known Zipf Law. While \( \beta=1 \), the gained votes of the political parties are inversely proportional with the respective rank. Here the eq. 1 can be denoted as frequency spectrum expressing the parties and the candidates \( N \) and the respective votes \( v \) [8],

\[
N(v) \propto v^{-\alpha}
\]  

(2)

Both can be represented statistically as probability to gain votes, \( v \), as:

\[
P_v(V = v) \sim v^{-\alpha}
\]  

(3)

where \( \alpha=1+1/\beta \) is power law exponent [9].

The three equations can be used to describe the general social phenomena. For instance, most of parties in Indonesian general election gain small amount of votes as to only few parties dominates the result of election.

The power laws in the results of election brings some consequences in our endeavor to understand the democratic process and how political preferences spreading through population. Generally speaking, the power law is related to properties in complex system and its criticality [1, 10]. Interaction among elements and some exogenous factors push the system to the critical conditions in which elements of the system self-organizing in such a way emerging robustness which is observable as long range correlation throughout time dimensionality. Elements become so responsive to the environment changes, for which any localities may give large impact to the whole system at large. It is easy to imagine the existence of newly coming political parties become majority in such robust socio-political system.

Ability of elements of complex system influence the system at large gives consequence of the strong interdependence among elements. Power law more likely cannot occur persistently if the social agents decide their choices independently. Thus, democratic process related to the independence of a choice for votes are dependent upon collective decisions and mutual interactions among agents.

Moreover, micro process among voters influencing each other and diffused throughout social network overcomes the dilemma faced by agents in any multi-party system. Social surroundings “guides” voters to optimize their choices as well as tendencies to rationally choose candidates or political party that more likely representing them. The boundedly rational agents [11] is thus balance their choices with the possible social tensions, since political votes sometimes act like a sort of social identities. In the other hand, political parties and candidates are also optimizing their performance in the continuum of being ideologically unique for the identities of the program being brought as well as accommodating as large citizens as possible in order to win the election. The dynamical interactions push the whole system toward its critical situation as well as driving the evolving political life in general.
Some other works related to opinion formation in social network has shown by Gonzalez et. al. showing the voting model of Sznajd being operated in pseudo fractal network emerging such interesting power law in Brazil and India [6]. This emphasizes the role of social network in voter’s choices giving new perspective on hierarchical network architecture of voters in a region with the phenomena of power laws in election result [5].

4. Concluding Remarks
Analysis to observe the survey on statistical properties of power laws in Indonesian legislative election in some scales (national, provincial, municipals, etc.) is brought. We have shown the interesting fact that the robustness of power laws are there for any aggregative levels. The scale free properties are shown everywhere while the various exponents of the power laws explain the heterogeneous microstructure emerging the power laws. It is worth to note that in special case of Indonesia, in the democratic General Elections of 1999 and the following 2004 and 2009 has unique patterns. The pattern between elections 2004 and 2009 is similar and both are far different with the statistical observation in General Election 1999. We have shown that in election 1999, there are two regimes of power laws expressing the unique political dynamics in this election with those following. The difference of mechanism and the micro social of the three elections are conjectured to be the main cause.

Conferring to the results in other countries of Mexico, Brazil, and India, an outline related to the observation on the heterogeneous choices in elections and performing candidates and political parties in the sense of complexity of socio-political landscape is drawn. A persistent self-organized system of political living in its running evolution. However, democracy should never have a final point.

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