Psychic embedding — vision and delusion

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Abstract

The paper introduces the idea that the human brain may apply complex mathematical modules in order to process and understand the world. We speculate that the substrate of what appears outwardly as intuition, or prophetic power, may be a mathematical apparatus such as time-delay embedding. In this context, predictive accuracy may be the reflection of an appropriate choice of the embedding parameters. We further put this in the perspective of mental illness, and search for the possible differences between good intuition and delusive ideation. We speculate that the task at which delusional schizophrenic patients falter is not necessarily of perception, but rather of model selection. Failure of the psychotic patient to correctly choose the embedding parameters may readily lead to misinterpretation of an accurate perception through an altered reconstructed of the object perceived.

1 Mathematics and the mind

Is mathematics an invention or a discovery? Should we perhaps look at it as an instrument invented by people to communicate abstract thoughts, tracing the evolution of language and conscience towards deeper subtleties of reasoning [47] [13]? Or should we instead regard it as a discovery of a natural ability of people to process and express information about nature and about themselves [31]?

Paul Erdős, although an atheist, spoke of an imaginary book in which God has written down the most beautiful mathematical proofs. For most of us, however, has mathematics remained in this mystical domain or — in our zeal to understand the world — have we changed it into a concrete and artificial produce, with elaborate but useless artwork and accessories? Has Man dug out its natural beauty only to transform it into a weapon and use it to tame and subdue the world?

In fact, nature is full of mathematical prodigy, from the perfection of the orange fruit to that of the honeycomb and of the spiderweb [25]. Plants and beasts compute as a genetic, unconscious necessity, and possibly so do we, mostly unknowingly. It is possible that, when we educate ourselves about mathematics, we lose the state in which we live it naturally and effortlessly.

Popular culture regards mathematics as the universal language of reason, used to communicate across cultures, time and even with alien civilizations. Socially, science has been gradually adopted as the unifying language of intellect. I believe however that, at its core, mathematics does not measure our “capacity to reason,” but quite the opposite. As much as contemporary science fiction [45] tries to persuade us that knowledge of elaborate and “magical” mathematical objects (such as prime numbers) is an evolved and uniquely reason-generated human feature [54], even plants can show this assumption wrong. And indeed, were it right — a newly arrived alien would innocently mistake a Rhododendron for the ruler of the Earth, deceived by its almost flawless use of number theory [25].
Autistic people are generally known to possess an extraordinary and inexplicable mathematical sense, which is sometimes concealed by the cognitive and social impediments inherent to the condition [57]. Sometimes, however, these attributes are so obvious that they not cease to amaze us, such as in the following two examples, well-known in the classical psychology literature. The case of the 1960s twin autistic savants was made famous by Oliver Sacks [44], particularly based on the boys’ ability to apprehend (somehow see) very large primes, although they were otherwise incapable of even the simplest computations. Almost at the opposite end of the autistic spectrum is the example of Temple Grandin, held to be the most accomplished and well-known adult with autism in the world. A Ph.D. in animal science and currently professor at Colorado State University, she solves exceptionally intricate design problems based on her ability to visualize and see the world in pictures [21]. According to one of her analogies, “When a mathematician becomes really skilled, he simply visualizes the abacus in his imagination and no longer needs a real one.”

Many creative people have mild forms of autism (Asperger’s syndrome), among which brilliant scientists; Albert Einstein would perhaps be diagnosed with Asperger if he lived today. Moreover, it is possible that we all have comparable abilities to some extent, but that in most cases they are so concealed by other brain mechanisms that we live our lives unaware of them. Here, an ethical quandary appears: what would a definitive cure mean in this context? Is total elimination of mental disease and of the consequent abilities really desired? Faced with this question during an interview [63], Temple Grandin presented herself as a proponent to neurodiversity, and did not support the idea of a cure for the entire autistic spectrum.

In this note I argue that “empathy,” or the legendary “psychic power” [35], is an example of a similar unconscious manifestation of intrinsic mathematical abilities of the human mind. Along history, people with access to unknown sources and with strangely accurate predictive abilities have been judged as charlatans and glorified as prophets, burned as sorcerers, but also locked up as mental patients. Over the next section, we will take a brief look at the definition, some history and a few examples of each of these instances. The following sections will focus more particularly on the relation between predictive powers, cognitive computational mechanisms and psychopathology.

2 Prophets and psychics

Originally, a prophet was considered a person who speaks for God or in the name of God, and who “carries God’s message to others”; the reception of a message was termed revelation and the delivery of this message was termed prophecy. In general, anyone considered prophetic could still be humanly fallible, could have erroneous opinions and could make inappropriate decisions, independently of the message transmitted or of the validity of the prophecies. Prophets are mentioned in many cultures throughout history, including Judaic, Persian, ancient Greek, Christian, Islamic and many others. According to the scriptures, Moses was commanded by God to deliver the Hebrews from slavery. Pythia, the Oracle of Delphi, spoke in riddles, but their interpretations had considerable influence within the Greek world, and the Oracle was consulted before all major personal and political decisions. Although in modern times the concept of prophecies has become somewhat looser and more controversial, claims of them have continued in many regions around the world. Particularly renowned is Michel de Nostredame (also known as Nostradamus), who was born in France at the beginning of the 16th century. He established his reputation as a “seer” (together with a world-wide and ageless collection of relentless critics) by publishing a collection of mysteriously-written prophecies of the major world events [41].
A more frequent reference in today’s popular culture is to the concept of “psychic,” which also implies an ability to perceive things hidden from the senses through extrasensory means. Despite intensive studies conducted by parapsychologists, to date there is no scientific evidence on the existence of such abilities, and skeptics have pointed out that most demonstrations of psychic occurrences are either trickery or self-delusion. However, belief in psychic phenomena is still widespread among the general public (a 2005 Gallup poll commissioned by Skeptical Inquirer concluded that 41% of Americans believe in extrasensory perception [59]). To consider only one famous example: Edgar Cayce has been referred to as a “prophet,” [51] a “mystic,” a “seer” and a clairvoyant. Among the abilities attributed to Cayce were producing advice on physical, mental and spiritual healing, astral projection, prophesying and mediumship. Cayce generally claimed not to remember what he had said while in the reading trance, when his unconscious mind had access to what his conscious mind did not.

Today, many people claim to have psychic abilities, and some make a living or earn celebrity on such grounds, making it at times hard to distinguish between failed good intentions and purposeful dishonesty [61]. While it may be easy to label all psychics as frauds, this may not necessarily be true. To start with, successful or not in their predictions, some people truly believe in their abilities (without just using them to claim fame or scam money), even when confronted with error in their readings. Such people are often classified or ostracized as delusional mental patients. Alison Smith [62] explains that future-telling is “categorised as a disease and not a gift because thus far there is no proof that any paranormal claim is valid.” The suggestion in this note could be regarded both along and against this view: when discussing delusional psychotic patients claiming divine revelations and prophetic power, I will rather view their qualities as a “prophetic strain” than as pure and inexplicable confabulation. Delusions of omniscience have been identified, differentiated, interpreted and contextualized along history in the most various ways.

Take again a most famous example. Jeanne d’Arc led the French army to several important victories during the Hundred Years’ War, claiming divine guidance, and she was canonized post-mortem as a saint in 1920. The consensus among scholars is now that her faith was sincere, and some Catholics regard her visions as truly divine inspiration [14]. Documents from her own era and historians prior to the twentieth century generally assume that she was both healthy and sane. A number of more recent scholars, however, have attempted to explain her visions in terms of psychiatric pathology, potential diagnoses including epilepsy, migraine, tuberculosis [40], and schizophrenia [16]. None of these hypotheses have gained consensual support, since no other characteristic symptoms were documented aside from Jeanne’s religious enthusiasm.

So is there any objective difference between Moses and Jeanne d’Arc, between Nostradamus and the psychic con artist? Or are the jury, the historical context and the interests at stake the only factors to decide towards one sentence or another?

3 Dynamics and prediction

The world exists as an unlimited dynamical process. Some people think of it as random [53], and associate events with the concept of “luck.” Other people emphasize its deterministic — although perhaps chaotic — character [23], and believe that its fate is pre-decided, or, more poetically put, “written in the stars.” We hereby chose to view the world as such an oversized deterministic system, governed by a set of underlying rules [34]. Knowledge of all these rules would automatically bring omniscience of the system, with its past, present and
future. The problems, however, of understanding such a system start even with the number of variables. Although some believe that the structural and functional complexity of the human brain are comparable with those of the Universe, the simultaneous perception of the world’s immense structure is incomprehensible for the human mind and the rationality of its science. Religious people attribute this capacity only to God or other higher power, and search for non-scientific ways of understanding this complexity. Without necessarily emphasizing the religious aspect, I suggest that, for grasping the world in its wholeness, such a “non-scientific” approach may not be a bad idea, but rather exactly the natural way for the brain to use its intrinsic toolbox.

Unable to simultaneously process the network or to ever know all its underlying rules, we attribute some events to chance, so that our “compromised” view of the world is a deterministic system infiltrated with noise — or, less optimistically put, a random system which exhibits deterministic “trends.” In such a system, God “does not throw dice” [17], but rather is the intractable deterministic source of apparent randomness. This is the view for instance in Isaac Asimov’s *Foundation* [2], in which a psychohistorian uses the mathematics of the field in order to weave through chaos and predict the downfall of an empire 300 years later.

One of the mathematical challenges of perception is that our senses and other sources of knowledge give us only a finite collection of discrete, time bounded measurements of the internal and external environments [3]. More precisely, our perception of the world’s evolution and occurrence of events throughout our lives is through a large, but finite collection of discrete measurements of a small number of variables over time: time series [50]. Based on time series, we research the past, understand change in time and constantly attempt to estimate the future and adapt to change, as part of our life-preservation effort. I hypothesize that the estimation of the properties of the world as a dynamical system involves a whole mental apparatus based on hard-core intrinsic mathematics. The correctness of this mathematical tool-box may be directly related to the accuracy of the information extraction from a seemingly noisy system — in particular to estimating the future of its dynamics. As in the case of the autistic twins unknowingly generating prime numbers in the manner of modern computers, people may make this computation unconsciously, as part of what we would call “intuition.”

We will henceforth refer to this intuitive ability as “psychic embedding,” since we believe that its underlying cognitive substrate involves a type of mathematical time-delay embedding (see next section). “Embedded cognition” (or “cognitive embedding”) might have been a more appropriate term, but the expression is already in use in a looser philosophical sense, in which the *embeddedness* refers to the modulation of the cognitive process that emerges from the interaction between the organism and the world [9] [12].

4 “To embed or not to embed?”

Time-delay embedding theorems provide a technique generally used to characterize the nonlinear behavior of a system, if enough information is provided by the observed time-series. Takens [52] showed in 1981 that it is mathematically possible to use such discrete information to predict the dimension of a possible attractor, while otherwise knowing nothing about the system’s laws and interactions.

Takens’ Theorem spurred a whole line of research focused on practical estimations of the evolution of real-world systems using time-delay embeddings. Aside from the quantity of the information obtained *a priori* (e.g., length and number of time series, which in the case of psychic embedding reflects perhaps the quality of perception), the accuracy of the
reconstruction based on time-delay embeddings is generally dramatically influenced by the choice of the embedding.

Indeed, when reconstructing from time-series data, one generally faces the problem of selecting a model (from within a model class [29] or between competing classes [26] [38]), usually in the form of estimating/fitting certain parameters to obey some criterion. In the case of time-delay embedding, the most natural constraints are choosing the embedding dimension and the time delay (or lag) so as to maximize either short-term or long-term prediction accuracy (thus generating either predictionist or behaviorist models, respectively). Kilminster et al. [29] offer an interesting example of both approaches applied to nonlinear circuit data, and further subject their results to a measure of prediction performance which they call ignorance.

Although this type of work has theoretically solved the problem of choosing the correct embedding dimension [52][46] and lag [27], applying the results on real data turned out to be hard; the process is in practice still very heuristic and far from being fully mastered. While the appropriate choice of these two parameters’ values has been accepted to play an important role in the performance of the model used for the time-series prediction, there are no longer “traditionally accepted” optimal choices and different researchers have proposed various analytical improvements of older estimates (see Appendix).

To make a long story short, parameter fitting for time-delay embeddings offers no warranties, and the process is still considered “more art than science” [55] [1]. In the context of psychic embedding, this could be interpreted as follows: finding the parameters that would equip us with reasonable short or long-term predictive qualities necessitates either a sequence of trial-and-error attempts, or, better, just good “intuition” of the embedder. This may be just the feature that assists our brain when working to extrapolate knowledge of today’s events to their future evolution and/or eventual conclusion. Some people never use their intuitive-embedding feature, and prefer to base their actions on pure reason, consciously deciding to shut down intuition and go by the “to do” list [7]. Other people act on it naturally and unconsciously. Some may blame the occasional positive predictive results on coincidence, but others, perhaps the ones with the most developed such embedding apparatus, get to a degree of accuracy which they can no longer deny, and decide to use this ability in a more controlled manner.

However, I further speculate that simple awareness or use of this machinery is not enough to generate accurate understanding of the system and reliable predictions, as illustrated — perhaps naively — by the following analogy. Most men in the village have a plow in the shed. Only some decide to take it out and use it, since they feel it is needed and appropriate to their life-styles and since they have the stamina to use it. Out of these, only a few will use it well and get good crops. Analogously, it is not enough to have the will to use the intricate machinery of system identification. Just as the inappropriate use of the plow would harm the field rather than make it fertile, in the same way a “broken” embedding machinery may lead to erroneous system identification and predictions. As illustrated in the Appendix, choosing the wrong context and goal for the embedding may render the result distorted and impossible to interpret correctly (see, for example [29] or [50] for illustrations of how different embedding parameters can affect the reconstruction outcome).

To end this section, we should make clear that time delay embedding is by no means the only possible substrate for psychic embedding, but rather just an illustrative example. As noted by Casdagli et al. [10], there are often clearly better alternatives to prediction. There are many approaches in the field of neural computation where the choice is left to the algorithm.
5 Embedding and mental illness

Deficits of logical reasoning, in particular delusions [4], have long been considered a hallmark of a whole collection of mental disorders. The delusional ideation is primarily affect-driven [36], and a mechanism that is present in healthy individuals when they are emotionally challenged may be inappropriately activated in delusional patients. Recent imaging data supports the hypothesis that the presence of delusions in medicated patients with schizophrenia is related to aberrant salience [43].

In a recent article, Kuperberg [30] reviews the known neuroanatomical imbalances found in delusional schizophrenics, from the inappropriate recruitment of the temporal and prefrontal areas when building meaning across sentences, to abnormal modulation within temporal and dorsolateral prefrontal cortices, leading to abnormal inferencing.

In our context, we can unify all these findings by interpreting them in relation to the underpinnings of the embedding process hypothesised in Section 4. It seems that, in the case of mental pathology (e.g. schizophrenia, delusional or bipolar disorder), the embedding hardware (neurons and their connections/functionality) have been impaired, either at the level of choosing the right embedding parameters, or at the stage of interpreting the end result, or perhaps at both these levels.

Maher et al. [33] have suggested that the psychopathology of delusions is a normal reaction to an abnormal but genuine perceptual experience. Indeed, other schizophrenic abnormalities may particularly encourage delusions (e.g., visual and auditory hallucinations are quite frequent in paranoid schizophrenia, and sometimes feed into intricate and intractable delusions). Here, however, we rather hypothesise that this does not happen necessarily (or only) directly as an act of impaired perception, but rather more indirectly, by interfering with the time-embedding process, or by clouding the interpretation of the result. It may be interesting to mention here the possible relation between embedding and the subjective experience of time, a fundamental constituent of human consciousness. The sense of time can be disturbed in mental disorders such as schizophrenia and affective disorders [56], with disturbances appearing not only at the level of past events retention (working memory), but also at the levels of the interference control and preparatory set [20]. It may no longer appear accidental in this context that Nostradamus has been speculated to “time-travel” in order to obtain his prophetic material [60].

In our interpretation, a delusional schizophrenic patient will not necessarily exhibit a perception deficit [8] or a sensory integration dysfunction (as in autism [5]), but rather a processing / interpreting abnormality that prevents him from contextualizing the input and from extrapolating it to a higher level of complexity. This is in agreement with another modern view of schizophrenia — as a (temporary or perpetual) incapacity of the brain to adapt its complexity to that of the changing environment [42]. In this context, estimating the complexity of both systems (the brain network and the perceptual world) may be of great interest and would put to great academic use similar embedding theorems with the ones used by the brain to realise this complexity adaptation (see Appendix). It is also interesting to put this view in the perspective of the Jasperian classification of three delusion subgroups: of perception, of idea or notion, and of awareness (Wahnstimmung) [24].

Most people pay no attention to unimportant, intractable “visions”, and continue unperturbed by such life details. The psychically-charged characters from Mircea Eliade’s Noaptea de Sanziene feel the “mystic” presence of a car, and anticipate its future disappearance in a accident a year from the present time [18]. B., A paranoid schizophrenic patient I once interviewed, had constructed a whole FBI story around the presence of a car on his street, and attempted suicide in order to escape the wrongly-predicted effects of what started as a genuine “gut instinct.” But who would blame him, when even biblical
prophets get lost in the translation interface and misinterpretation?

Many people believe in the existence of a higher power. Not everybody, however, claims to literally hear the voice of God addressing him directly with requests and promises. Indeed, religion is an enduring theme in psychosis [6]. Ng [39] conceptualises psychosis as “the manifestation of aberrant perceptual and integrative processes.” The prevalence of religious themes among psychotic patients may be explained by its cultural role, by the involvement of temporolimbic overactivity in the pathogenesis of psychosis, and the tendency to interpret intense or discrepant events as spiritual [39]. How then can we ever distinguish between religion as a culture and religiosity as pathology? Interestingly, a recent study [15] analyzed the content of religious visions/delusions, and concluded that in mythological narratives, God appears showing his back as departing, while psychiatric patients experience the divinity in a direct way.

It may happen (although rarely) that the voice of reason comes to conclusions which are openly contradictory to the ones of the delusion [19]. For example, I once discovered with great surprise that B., the paranoid patient mentioned earlier, was aware that his FBI paranoia might be a delusion, but could still not stop the emotional effects of this idea. That may well be because the parallel conscious process involves different cortical pathways from the intuition-based delusion.

We suggest that one could obtain more testable theories on the pathways and mechanisms of psychic embedding by using cognitive and imaging paradigms — such as recording brain activity of healthy subjects and delusional patients while guessing the outcome of a real event presented through visual or other cognitive clues.

6 Conclusions

In the perspective of this paper, it may not be just a linguistic coincidence that the words “intuitive” and “seer” are used often times instead of “psychic” or “empath”. It is also remarkable that schizophrenic patients are known to be exceptionally warm and “empathetic” people, naturally (if not purposefully) turmoiled with the emotions of themselves and others. I gradually came to believe that delusional patients are some sort of failed prophets, who have the gift, but do not have the ideal means to use it — much in the manner of autistic/Asperger patients, who have great geometric vision, but often lack the cognitive strength to even express it.

In this context, a cure which would eliminate mental disease would not mean reducing mankind to a homogeneous, “normal” mass, but rather better emphasizing each person’s potential. It would not mean turning every timer into a silent Swiss watch, but rather repairing the broken clock so that it would again play its beautiful tune when striking the hours.

Appendix

In a deterministic system, we expect in general that each time value of any variable $X_t$ at time $t$ depends on a finite number of states, i.e. there exist a function $f$ and a memory horizon $k$ such that $X_t = f(X_{t-1}, X_{t-2}, ..., X_{t-k})$. When studying the topology or dynamics of such a system, one may try to capture them into one or several quantifiable invariants (such as the set of the system’s Lyapunov exponents, the topological or Kolmogorov entropy, or — in the case an attractor exists — the attractor’s dimension and geometry). A most remarkable feature of chaotic data is that this type of information is in principle encompassed in the evolution of any one variable. This property is crucial, since many
high dimensional physical systems are often observable through the time lags of a single scalar variable (observable [52]). Embedding techniques extracts this type of information by working in a space of appropriate dimension (e.g., in which the attractor can be embedded), thus avoiding undesirable full system reconstruction. The method was initially proposed by Packard and Takens [52], and later continued by Sauer et al. [46].

From a scalar time series \{X_t\}_{t=1}^N of N observations of the variable X, the method of time-delay embedding is used to reconstruct a vector time series with evolution topologically equivalent to the original system, through the transformation:

\[ X_t \rightarrow (X_t, X_{t-\tau}, X_{t-2\tau}, ..., X_{t-(d-1)\tau}) \]

where the lag \( \tau \) relates to underlying time scales in the time series. The embedding dimension \( d \) can be estimated using geometric methods such as false nearest neighbours [28]. Although some researchers have argued that the critical parameter is the product \( d\tau \) [11], in general it is desirable to calculate both parameters \( d \) and \( \tau \). Unfortunately, the quantity of data required for correct computations can be quite substantial. For example, an estimate by Sprott [50] suggests that approximately \( 10^{2+0.4D} \) datapoints are necessary to accurately reconstruct an attractor of dimension \( D \).

The field is undergoing a constant effort to find optimizations of the existing methods. Small et al. [48] [49], for example, propose an extension of the classical algorithm based on strategies introduced by Judd and Mees [26], which has many advantages over the standard techniques. Instead of finding the two parameters \( d \) and \( \tau \), they search for the embedding window \( d_w \) and for a whole set of adjustable time lags \( 1 \leq l_1 \leq l_2 \leq ... \leq l_k \leq d_w \) to construct the non-uniform time delay embedding

\[ X_t \rightarrow (X_t, X_{t-l_1}, X_{t-l_2}, ..., X_{t-l_k}) \]

On one hand, optimizations of old results are aimed towards constructing more correct or faster algorithms to compute the invariants using delay embedding [32] [22] [58]. On the other hand, some studies focus on obtaining the most useful predictions from shorter and noisier time series [37], since experimental data usually presents such constrains. For noisy data, the choice of the embedding parameters can make a big difference on the success of the prediction algorithm [29].

References


