THE ALEPH ZERO OR ZERO DICHOTOMY

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Abstract. The Aleph Zero or Zero Dichotomy is a strong version of Zeno's Dichotomy II which being entirely derived from the topological successiveness of the ω^*-order comes to the same Zeno's absurdity.

1. Zeno's paradoxes and modern science

Zeno's Paradoxes have interested philosophers of all times (see [13], [14], [79], [68], [47] or [24] for historical background), although until the middle of the XIX century they were frequently considered as mere sophisms [13], [14], [67], [68]. From that time, and particularly along the XX century, they became the unending source of new philosophical, mathematical and physical discussions. Authors as Hegel [43], James [48], Russell [67], Whitehead [81], [82] or Bergson [9], [10] focused their attention on the challenging world of Zeno's paradoxes. At the beginning of the second half of the XX century the pioneering works of Black [11], Wisdom [83], Thomson [75], [76], and Benacerraf [8] introduced a new way of discussing the possibilities to perform an actual infinity of actions in a finite time (a performance which is involved in most of Zeno's paradoxes). I refer to Supertask Theory [64]. In fact, infinity machines, or supermachines, are our modern Achilles substitutes. A supermachine is a theoretical device supposedly capable of performing countably many actions in a finite interval of time. The possibilities of performing an uncountable infinity of actions were ruled out by P. Clark and S. Read [22], for which they made use of a Cantor's argument on the impossibility of dividing a real interval into uncountably many adjacent parts [19]. Although supertasks have also been examined from the perspective of nonstandard analysis ([55], [54], [1], [52]), as far as I know the possibilities to perform an hypertask along an hyperreal interval of time have not been discussed, although finite hyperreal intervals can be divided into uncountable many successive infinitesimal intervals, the so called hyperfinite partitions ([73], [34], [49], [44], etc.). Supertask theory has finally turned its attention, particularly from the last decade of the XX century, towards the discussion of the physical plausibility of supertasks ([66], [60], [64], [68], [39], [41], [40]) as well as on the implications of supertasks in the physical world ([60], [61], [62], [30], [63], [58], [2], [3], [65]), including relativistic and quantum mechanics perspectives [80], [45], [28], [29], [58], [27], [70]

During the last half of the XX century several solutions to some of Zeno's paradoxes have been proposed. Most of these solutions were found in the context of new branches of mathematics as Cantor's transfinite arithmetic, topology, measure theory [37], [38], [85], [39], [41], [40], and more recently internal set theory (a branch of nonstandard analysis) [55], [54]. It is also worth noting the solutions proposed by P. Lynds within a classical and quantum mechanics framework [51]. Some of these solutions, however, have been contested [59], [1]. And in most of cases the proposed solutions do not explain where Zeno's arguments fail [59], [64]. Moreover, some of the proposed solutions gave rise to a significant collection of new and exciting problems [68], [47] [70].

The four most famous paradoxes of Zeno are usually regarded as arguments against motion ([4], ([38], [42], [23], [68] etc.) be it performed in a continuous or in a discontinuous world. Achilles and the Tortoise and the Dichotomy in the continuous case, the Stadium and the Arrow in the discontinuous one. The paradoxes of the second case (together with the paradox of Plurality) are more difficult to solve, if a solution exists after all, particularly in a quantum spacetime framework. Most of the proposed solutions to Zeno's paradoxes are, in effect, solutions to the paradoxes of the first group or to the second one in a dense continuous spacetime framework. This situation is very significant taking into account the increasing number of contemporary physical theories suggesting the quantum nature of spacetime, as for instance Superstring Theory ([35], [36] [77], [31]), Loop Quantum Gravity ([71], [5] [72]), Quantum Computation Theory ([74], [6], [50]) or Black Hole Thermodynamics [6], [74]. Is, therefore, at this quantum level where physics (the science of changes) will finally meet the problem of Change [7] whose insolvability probably motivated Zeno's arguments? Is the problem of Change really inconsistent as some authors ([56], [57]) claimed? These are in fact two intriguing and still unsolved questions related to Zeno's arguments [59].

2. Zeno's paradoxes and the ω-order

No less intriguing, though for different reasons, is the fact that one immediately perceives when examining the contemporary discussions on Zeno's paradoxes. Surprisingly, the Axiom of Infinity is never involved in such discussions. Zeno's arguments have never been used to question the Axiom of Infinity, as if the existence of actual infinite totalities were beyond any doubt [32]. Grünbaum, for instance, proposed in this sense that if it were the case that from modern kinematics together with the denseness postulate a false zenonian conclusion could be formally derived, then we would have to replace kinematics by other mechanical theory [38]. Anything but questioning the hypothesis of the actual infinity from which topological denseness ultimately derives. And this in spite of the lack of selfevidence of that hypothesis, which is even rejected by some schools of contemporary mathematics as constructivism (among whose precursors we find scholars as Newton, Fermat or Euler [53]).

In the second half of the XIX century B. Bolzano [12] and R. Dedekind [26] tried unsuccessfully to prove the existence of actual infinite totalities. For his part, G. Cantor, the founder of transfinite mathematics, simply took it for granted. Thus, in §6 of his famous *Beiträge* (pp. 103-104 of the English translation [18]) we can read:

> The first example of a transfinite set is given by the totality of finite cardinal numbers ν.

although, as could be expected, he gave no proof of that existence. In accordance with his profound theological platonism [25], Cantor was firmly convinced of the actual existence of infinite totalities [16], [17], [15], [20], [21]. But convictions do not suffice in mathematics and finally we had to state that existence by the expeditious way of axioms.

The cantorian notion of ω-order is an immediate consequence of assuming the set of finite cardinals as a complete totality ([18], p. 115):

> By ω we understand the type of a well ordered aggregate
>
> $$(e_1, e_2, \dots, e_\nu, \dots)$$

in which

$$e_\nu \prec e_{\nu+1}$$

and where ν represents all finite cardinal numbers in turn.

In modern terms, we say that a sequence is ω-ordered if it has a first element and each element has an immediate successor. Similarly a sequence is ω^*-ordered if it has a last element and each element has an immediate predecessor. Evidently, both types of ordering are intimately related to most of Zeno's arguments particularly to both dichotomies, although, surprisingly, the analysis of Zeno's arguments as formal consequences of the ω-ordering remains still undone. For some unknown reasons, it seems we are not interested in analyzing the formal consequences of assuming the existence of sequences (lists) which are simultaneously complete and uncompletable, as is the case of both the ω-ordered and the ω^*-ordered sequences (they are in fact complete because this is what the Axiom of Infinity states, and uncompletable because there is not a last (first) element which complete them). No matter the enormous problems the actual infinity means for experimental sciences as physics (recall for example the problems of renormalization in particle physics [33], [46], [35], [84], [36], [69]).

Apart from discussing the nature of motion and some others philosophical subtleties, Zeno's argument can also be used to question the formal consistency of the actual infinity. The short discussion that follows is just oriented in that direction. Its main objective is to analyze a version of Zeno's Dichotomy II based on the topological successiveness of the ω^*-ordered sequences of real numbers within any real interval. The result leads to a dichotomy, the Aleph Zero or Zero Dichotomy, whose formal consequence coincides with Zeno's absurdity, although in this case it is formally derived from the Axiom of Infinity via the ω-ordering.

3. The Aleph Zero or Zero Dichotomy

Let us consider the famous Achilles' race rightward along the X axis from point 0 to point 1 whose impossibility Zeno's Dichotomy II claims. In the place of the uncountable and densely ordered sequence of points within the real interval [0, 1] we will only consider the ω^*-ordered sequence of points:

$$\ldots, \frac{1}{2^4}, \frac{1}{2^3}, \frac{1}{2^2}, \frac{1}{2}, 1 \tag{1}$$

Achilles must successively traverse at a finite uniform velocity v in order to reach point 1 starting from point 0. In fact this denumerable sequence of points (Z^*-points according to classical Vlastos' terminology [78]) is not densely but successively ordered, which means that between any two successive Z^*-points no other Z^*-point exists. In consequence, and at a finite velocity, Z^*-points can only be traversed in a successive way: one after the other. Due to the topological denseness of the real number continuum modelling space and time, no instant has an immediate successor instant in the same way that, for instance, natural number have. On the contrary, between any two instants uncountably many other instants exist. Consequently the last instant at which Achilles is still at rest is not followed by the first instant at which Achilles is already running. Thus, although it will irrelevant for the discussion that follows, we must decide if t_0 is the last instant at which Achilles is still at rest or it is the first instant at which he is running, although in this last case we have also to assume that at t_0 Achilles has traversed a zero distance (otherwise he would be running at an infinite velocity). This said, we will assume that t_0 is the last instant at which Achilles is still at rest. According to classic mechanics Achilles will reach point 1 just at $t_1 = t_0 + 1/v$. But before reaching his goal, he has to successively traverse the controversial Z^*-points. We will focus our attention just on the way Achilles performs such a traversal. For this, let $f(t)$ be the number of Z^*-points Achille has traversed at the precise instant t, being t any instant within the closed interval $[t_0, t_1]$. It is quite clear that $f(t_0) = 0$ because at t_0 Achilles has not begun to run. For any other instant t within the half closed interval $(t_0, t_1]$ Achilles has already passed over countably many Z^*-points, for if there were an instant t in $(t_0, t_1]$ at which Achilles has only passed over a finite number n of Z^*-points, these n Z^*-points would have to be the impossible firsts n points of an ω^*-ordered sequence of points. So we can write:

$$f(t) = \begin{cases} 0 & \text{if } t = t_0 \\ \aleph_0 & \text{if } t_0 < t \leq t_1 \end{cases} \tag{2}$$

Notice $f(t)$ is well defined for each t in $[t_0, t_1]$. Consequently, f maps the real interval $[t_0, t_1]$ into the set of two elements $\{0, \aleph_0\}$. In this way f defines a dichotomy, the Aleph Zero or Zero Dichotomy, regarding the number of Z^*-points Achilles has traversed when moving rightward from 0 to 1 along the X axis. Accordingly, with respect to the number of the traversed Z^*-points, Achilles can only exhibit two states:

(1) State A_0: Achilles has traversed 0 Z^*-points.
(2) State $A_{\aleph_0}$: Achilles has traversed $\aleph_0$ Z^*-points.

Thus, Achilles directly becomes from having traversed no Z^*-point (state A_0) to having traversed $\aleph_0$ of them (state $A_{\aleph_0}$). Finite intermediate states, as A_n at which Achilles would have traversed only a finite number n of Z^*-points, simply do no exist.

Let us now examine the transition from A_0 to $A_{\aleph_0}$ under the inevitable restriction of the Aleph Zero or Zero Dichotomy. The topological successiveness of Z^*-points makes it impossible that they can be traversed other than successively. And taking into account that between any two successive Z^*-points a finite distance greater than zero exists, to traverse $\aleph_0$ successive Z^*-points -whatever they be- means to traverse a finite distance greater than 0. This traversal, at the finite Achilles' velocity, can only be accomplished by lasting a certain amount of time necessarily greater than 0. Achilles, therefore, has to expend a certain time $\tau > 0$ in becoming $A_{\aleph_0}$ from A_0. This time τ is indeterminable, otherwise we would know the precise instant at which Achilles becomes $A_{\aleph_0}$ and, consequently, we would also know the precise Z^*-point on which he reaches that condition, which is evidently impossible because in this case there would have to be a natural number n such that $n+1 = \aleph_0$. The indeterminacy of τ means both the existence of more than one alternative for its value and the impossibility to determine the precise alternative. Now then, indeterminable as it may be, τ has also to be greater than 0 and this requirement is incompatible with the Aleph Zero or Zero Dichotomy. In effect, let r be any real number greater than 0. It is immediate to prove that r is not a valid value for τ because if that were the case we would have:

$$\forall t \in (0, r): \; 0 < f(t) < \aleph_0 \tag{3}$$

going against the Aleph Zero or Zero Dichotomy. Therefore it impossible for τ to be greater than 0, which in addition is confirmed by the inexistence of finite intermediate states A_n. Consequently, Achilles cannot become $A_{\aleph_0}$ at his finite velocity v. He must therefore remain A_0. Or in other words, he cannot begin to move. Evidently, this conclusion is the same absurdity claimed by Zeno's Dichotomy II, although in our case it has been entirely derived from the topological successiveness of the ω^*-order, which in turns derives from the assumed existence of *complete* denumerable totalities (actual infinities) [18], i.e. from the Axiom of Infinity. It is therefore this axiom the ultimate cause of the above Zeno's absurdity.

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