The routing function of ALD is deadlock-free. The proof is presented as follows:

Because of the cycle detection mechanism, the channel dependency graph G of the s-VC subnetwork is traffic-configuration-dependent. But for any certain configuration, the channel dependency graph can be generated without ambiguity.

(1) First, we show that for any certain configuration, G is acyclic. From the quadrant-based load-balance scheme, the message is forwarded unidirectionally along each dimension, so there cannot be any dependency between s-VCs along different directions of the same dimension. Because of the dimension-order feature, there cannot be any cycle involving s-VCs along different dimensions. And from the design of cycle detectors, there cannot be any cycle among s-VCs belonging to the same s-VC ring along any dimension. Therefore, G is acyclic.

(2) Furthermore, we show that no s-VC can ever participate in a deadlock by using induction on the dimension index. For presentation convenience, an n-dimensional torus network is assumed, whose s-VC subnetwork implements the DOR routing with the dimension order X_0, X_1, …, X_{n-1}.

- *Basis*: Consider the s-VCs on an s-VC ring R^s, which belongs to a ring R of the physical channels along dimension X_{n-1}. Suppose there is a deadlock configuration in the network, and a message m with destination y is deadlocked such that one of its flits is on an s-VC of R^s, $c^s(m)$. Because of the dimension-order feature, the header flit of m must be on the ring R, and will be forwarded along R. No matter it is now on an s-VC or an n-VC, the header flit can certainly be accessed into an s-VC of R^s, $c'^s(m)$, since m has already been registered to the cycle detector of R. So there is a channel dependency from $c^s(m)$ to $c'^s(m)$.

  If m is the only message that has its flit on the s-VC of R^s, obviously its header can be forwarded in the s-VCs, and is not deadlocked. Suppose there are totally k messages of that kind. Because G is acyclic, these k messages can be indexed from 1 to k in the way that $i < j$, if there is a dependency from $c'^s(m_i)$ to any s-VC on R^s occupied now by m_j. Then there is no dependency from $c'^s(m_k)$ to any s-VC occupied by other messages, so that the header flit of m_k can be forwarded in the s-VCs, and is not deadlocked. Therefore, no s-VC along dimension X_{n-1} can ever participate in a deadlock.

- *Inductive Step*: We show that no s-VC along dimension X_s can ever participate in a deadlock, if no s-VC along dimension X_t cannot, for any $t > s$. Consider the s-VCs along dimension X_s. Suppose there is a deadlock configuration in the network, and a message m with destination y is deadlocked such that one of its flits is on an s-VC along dimension X_s. If the header flit of m requires the output VC along dimension X_t, $t > s$, it can be accessed into the s-VCs and forwarded, since no s-VC along dimension X_t participates in the deadlock. So *m* is not deadlocked. If the header flits of all the deadlocked messages require the output VCs along dimension X_s, the situation is reduced to a ring, and the remaining of the proof is just the same with what is done for dimension $X_{n\text{-}1}$ in the *Basis*. Consequently, no s-VC along dimension X_s can ever participate in a deadlock.

Therefore, no s-VC in the torus network can ever participate in a deadlock.

(3) Suppose there is a deadlock configuration, from (2), no s-VC is occupied by the deadlocked messages, so that the deadlocked messages can be accessed into the s-VCs and forwarded. Therefore, there is no deadlock.