A Polar Concept Argument for the Existence of Abstracta

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ABSTRACT I present a polar concept argument for the existence of abstract objects. After recalling the fundamentals of the debate about the existence of abstracta, I present in a detailed way the argument for the existence of abstracta. I offer two different variations of the argument: one, deductive and the other, inductive. The argument rests primarily on the fact that our universe is well-balanced. This well-balanced property results from the fact that all instantiable polar dualities are instantiated. Hence, the abstract pole of the abstract/concrete duality must also be exemplified. Lastly, I review several objections that can be raised against the present argument.

There are several famous problems about abstract entities. One of them consists of whether there exist any abstract objects. A second issue is concerned with the definition of which sorts of entities are genuinely abstract. A third issue relates to whether the abstract/concrete duality is exhaustive or not. The purpose of this paper is to address the first of these issues and to describe a polar concept argument that entails the existence of abstracta. Before stating the argument in detail and reviewing several objections that can be raised against it, it is worth recalling preliminarily the fundamentals of the debate about the existence of abstracta.

1. The debate about the existence of abstracta

Let us recall preliminarily the main lines of the issue of whether there exist abstract objects[1]. This latter problem rests primarily on the abstract/concrete distinction. Uncontroversially, the following objects are considered as abstracta: the natural numbers, the cosine function, the Pythagorean theorem. For this reason, I shall only be concerned in what follows with paradigm abstracta, i.e. natural numbers, setting aside other sorts of entities whose status remains controversial. On the other hand, an instance of a jay or of an oak-tree, the mountain in front of me, the sun, our galaxy, are paradigm concrete objects. Uncontroversially, concreta are considered as existent objects. But at this stage, agreement stops. In effect, by contrast, the mere existence of abstracta is at issue. Do abstract objects truly exist? There are two main philosophical answers to this question. On the one hand, some philosophers simply deny the existence of abstracta. According to the corresponding line of thought, only concrete objects exist in our universe, and abstract concepts are a mere product of our brain circuitry. Thus, natural numbers, the sine function, etc., which are considered as paradigm cases of abstracta, exist only in our mind. The view that denies the existence of abstract objets is *nominalism*.

On the other hand, according to a line of thought originating from Plato, abstract objects do truly exist. According to *platonism*, abstracta have an existence of their own, which parallels the existence of concreta. In addition, abstracta are classically considered as having no spatiotemporal position, in contrast to concreta which possess a given position in space and time.

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[1] E. J. Lowe (1995) distinguishes three different conceptions of abstract objects. In what follows, my concern will be with what he terms abstract$_1$ objects, i.e. in opposition to concrete objects.

The main argument for abstracta is the Quine-Putnam *indispensability argument*.[2] According to its first premise, we should be committed to the existence of all entities that are indispensable to our best scientific theories. Given that natural numbers are indispensable to these scientific theories, it follows that we should be committed to the existence of natural numbers. The indispensability argument is controversial and has notably led to important objections raised by Hartry Field (1980), Penelope Maddy (1992) and Elliott Sober (1993). Without entering into the details of these criticisms, I will offer here a different line of argument for the existence of abstract objects.

2. The polar concept argument for the existence of abstracta

In what follows, I borrow the expression 'polar concept argument' from the characterisation of Ryle's argument against scepticism[3] (1960), put forth by Anthony Grayling (1995, pp. 49-50). Grayling describes Ryle's argument in the following terms:

> The point can be simply illustrated by a consideration of Gilbert Ryle's attempt to refute the argument from error by a 'polar concept' argument. There cannot be counterfeit coins, Ryle observes, unless there are genuine ones, nor crooked paths unless there are straight paths, nor tall men unless there are short men. Many concepts come in pairs which are polar opposites of one another, and these conceptual polarities are such that one cannot grasp either pole unless one grasps its opposite at the same time. Now error and 'getting it right' are conceptual polarities. If one understands the concept of error, one understands the concept of getting it right. But to understand this latter concept is to be able to apply it. So our very grasp of the concept of error implies that we sometimes get things right.

Grayling characterises thus as a *polar concept argument* the argument used by Gilbert Ryle to refute the argument from error, which arises in the context of the debate against scepticism. The argument from error puts in parallel two types of situations. On the one hand, it appears that we often mistakenly have some knowledge that comes from perceptual experience. But these latter situations, the sceptic argues, are indistinguishable from present situations in which we have some knowledge that stem from our current perceptual experiences. Therefore, concludes the sceptic, our present knowledge could also be mistaken. According to Ryle, the argument from error is inconclusive, since 'getting it right' and 'error' are opposites and originate from the same duality. Hence, Ryle pursues, whenever one grasps the concept of 'error', one also grasps the opposite concept of 'getting it right'. A further step states that to understand the corresponding concept is tantamount to being able to apply it in practice. And this finally undermines the conclusion of the argument from error. I shall not discuss here whether Ryle's argument is valid or not. Rather, my concern will be with showing that a polar concept argument along the same lines can be used in support of the existence of abstracta.

The polar concept argument for the existence of abstracta can be sketched informally as follows. Begin with the fact that our universe is well-balanced. A rough analysis reveals that this well-balanced property is exemplified many times. Consider for example the existence, at a macroscopic level, of very large objects such as stars, supernovae or galaxies. Contrast now with the existence, at a microscopic level, of very small objects such as atoms or molecules. This illustrates how our universe is well-balanced with regard to the large/small duality. Now it appears that numerous other polar opposites are also instantiated in our universe. Consider then how both poles of the hot/cold duality are exemplified. It suffices to consider the existence, on the one hand, of hot objects such as stars, and

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[2] For a survey of the *indispensability argument*, see Colyvan (2001).
[3] Cf. Ryle (1960, pp. 94-95): "A country which had no coinage would offer no scope to counterfeiters. There would be nothing for them to manufacture or pass counterfeits of. They could, if they wished, manufacture and give away decorated disks of brass or lead, which the public might be pleased to get. But these would not be false coins. There can be false coins only where there are coins made of the proper materials by the proper authorities. In a country where there is a coinage, false coins can be manufactured and passed; and the counterfeiting might be so efficient that an ordinary citizen, unable to tell which were false and which were genuine coins, might become suspicious of the genuineness of any particular coin that he received. But however general his suspicions might be, there remains one proposition which he cannot entertain, the proposition, namely, that it is possible that all coins are counterfeits. For there must be an answer to the question `Counterfeits of what?'".

on the other hand, of cold objects such as brown dwarfs, dead stars or asteroids. This shows that our universe is also well-balanced with regard to the hot/cold duality. Now consider how many dualities such as attractive/repulsive, static/dynamic, bright/dark, positive/negative, neutral/charged, visible/invisible, etc., are also instantiated. At this step, it is worth considering the case of the abstract/concrete duality. There is uncontroversial evidence that *concrete* objects do exist in our universe. Now it follows that the necessary well-balance of our universe with regard to the abstract/concrete duality also entails the necessary existence of abstract objects.

At this step, for the sake of accuracy, it is worth stating a few definitions. Let us begin with *polar opposites*. Intuitive though it is, the notion of polar opposites needs in effect clarifying. Paradigm examples of polar opposites are positive/negative, small/large, static/dynamic, internal/external, etc. But let us provide an explicit definition. To begin with, *polar opposites* are *polar concepts*, i.e. concepts which intuitively come in pairs (let us term them A and Ā), and are such that each one is defined as the opposite of the other. For example, *internal* can be defined as the opposite of *external*, while symmetrically, *external* can be defined as the opposite of *internal*. Both poles are the contrary of one another. In a sense, there is no primitive notion and neither poles of the A/Ā duality can be regarded as the primitive notion.

Let us stress, second, that both poles of a given duality are *simple* qualities, in opposition to composite qualities. The distinction between simple and composite qualities can be drawn as follows. Let A_1, A_2 be simple qualities. Now $A_1 \wedge A_2$, $A_1 \vee A_2$ are composite qualities. To give an example, *small*, *static*, *positive* are simple qualities, while *small and static*, *small and positive*, *static and positive*, *small and static and positive* are composite qualities. A more general definition is then as follows: B_1, B_2 being simple or composite qualities, $B_1 \wedge B_2$, $B_1 \vee B_2$ are composite qualities. Incidentally, this also casts light on the reason why red/non-red, blue/non-blue cannot be considered as polar opposites. For example, *non-red* can be defined as *violet* $\vee$ *indigo* $\vee$ *blue* $\vee$ *green* $\vee$ *yellow* $\vee$ *orange* $\vee$ *white* $\vee$ *black*.

It is worth mentioning, third, that polar opposites are *neutral* concepts, i.e. neither meliorative nor pejorative. Accordingly, *large*, *small*, *external*, *internal*, *concrete*, *abstract*, etc., are neutral polar concepts, to the difference of such concepts as *beautiful*, *ugly*, *courageous*, which are non-neutral.

Given this definition, we are notably in a position to distinguish polar opposites from *vague* objects. It should be noticed first that polar opposites and vague objects share some common features. In effect, vague objects come in pairs, in the same way as polar opposites. In addition, vague concepts are classically viewed as possessing an extension and an anti-extension, which are mutually exclusive. Such a feature is also shared by polar opposites. But let us stress now the differences between the two categories. A first difference (i) consists in the fact that the extension and the anti-extension of vague concepts are not jointly exhaustive, for they admit of borderline cases (and also borderline cases of borderlines cases, etc., thus giving rise to the hierarchy of higher-order vagueness), which constitute a penumbral zone. In contrast however, polar contraries do not necessarily possess this latter feature. In effect, polar opposites can be either exhaustive or non-exhaustive. For example, the abstract/concrete duality is intuitively exhaustive, for there does not seem to exist objects that are neither abstract nor concrete. Now the same goes for the vague/precise duality: intuitively, there does not exist objects which are neither precise nor vague and pertain to an intermediate category. Hence, there exists polar opposites which are not vague, e.g. both poles of the abstract/concrete duality. Now a second difference (ii) between polar opposites and vague objects is that the former are simple qualities, while the latter consist of either simple or composite qualities. For there exists so-called multi-dimensional vague concepts, such as *vehicle*, *machine*.[4] Lastly, a third difference (iii) resides in the fact that some polar opposites are inherently precise. To take an obvious example, the positive/negative duality is entirely made up of precise constituents.

Let us also distinguish, second, polar contraries from the pair consisting of a given concept and its complement. To take an example, positive/negative are polar opposites, to the difference of positive/non-positive. For *non-positive* includes both *neutral* and *negative*. In the same way, negative/non-negative are not genuine polar opposites, since *non-negative* includes both *neutral* and *positive*. However, if a given duality A/Ā is exhaustive, it follows that the polar opposite of A (respectively Ā) identifies itself with non-A (respectively non-Ā). But as we have seen, not all polar

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[4] From Soames (1999, p. 217).

dualities are exhaustive and this entails that the polar opposite of a given concept must be distinguished, from a general viewpoint, from its complement.

On the other hand, it is also worth defining the *well-balanced property*. For a given object *o* of which one part has a property A, the well-balanced property relative to the A/Ā duality results from the fact that there also exists another part of *o* which has the opposite property Ā. To give an example, protons have a positive charge, while on the other hand, electrons have a negative charge. Thus, an atom of hydrogen, which includes one electron and one proton, is well-balanced with regard to the positive/negative duality, since it has both a positive and a negative charge. More generally, being well-balanced relative to our universe results from a generalisation of this latter property to all polar dualities.

At this step, we are in a position to state the present argument more explicitly:

(1)	our universe is well-balanced	premise
(2)	in our universe the following polar dualities are instantiated: large/small, positive/negative, external/internal, absolute/relative, bright/dark, static/dynamic, attractive/repulsive, visible/invisible, cold/hot, etc.	evidence
(3)	the well-balanced property relative to the A/Ā duality results from the fact, for a given object having a polar property A, of also having the opposite property Ā	definition
(4)	∴the well-balance of our universe results from the fact that all instantiable polar opposites are instantiated	from (1),(2),(3)
(5)	*concrete* pertains to the abstract/concrete duality	definition
(6)	concrete objects exists in our universe	evidence
(7)	the concrete pole of the abstract/concrete duality is instantiated	from (6)
(8)	the abstract pole of the abstract/concrete duality is necessarily instantiated	from (3),(4),(7)
(9)	∴ abstract objects exists in our universe	from (8)

The argument being stated, it is worth highlighting now some of its distinctive features. It should be pointed out first that the argument is deductive. In effect, it starts from the consideration that our universe is well-balanced and derives the conclusion that abstract objects do exist. The well-balanced property is crucial to the argument and two different types of well-balanced properties can be distinguished: (i) well-balanced relative to a given polar duality A/Ā; (ii) well-balanced relative to our universe.

It is also useful to define accurately the scope of the argument. More generally, the argument postulates that for each pole observed in our universe, there exists an opposite pole. The argument is thus based on the fact that for every object that *exists* and exemplifies a pole, there also exist in our universe other objects that instantiate the opposite pole. The argument postulates that there do not exist things in our universe that instantiate a pole of a given duality without also instantiating the opposite pole.

Lastly, it should be pointed out that the above argument could be stated alternatively under the form of an *inductive* argument. It would then be recast as follows:

(1i)	in our universe the following polar dualities are instantiated: large/small, positive/negative, external/internal, absolute/relative, bright/dark, static/dynamic, attractive/repulsive, visible/invisible, cold/hot, etc.	evidence
(2i)	the well-balanced property relative to the A/Ā duality results from the fact, for a given object having a polar property A, of also having the opposite property Ā	definition
(3i)	∴in our universe all instantiable polar dualities are instantiated	from (1i), induction
(4i)	the well-balance of our universe results from the fact that that all instantiable polar opposites are instantiated	from (1i),(2i),(3i)
(5i)	*concrete* pertains to the abstract/concrete duality	definition
(6i)	concrete objects exists in our universe	evidence
(7i)	the concrete pole of the abstract/concrete duality is instantiated	from (6i)

(8i)   the abstract pole of the abstract/concrete duality is necessarily instantiated   from (2i),(4i),(7i)

(9i)   ∴ abstract objects exists in our universe   from (3i),(4i),(5i)

It should be noticed that the inductive form of the argument proceeds by enumerating all instantiable polar dualities and then generalising to all polar dualities. It follows then straightforwardly by induction that the abstract/concrete duality is also instantiated.

3. Response to objections

At this stage, it is worth considering several objections that can be pressed against the present argument. Let us review, first, a line of objection that stems from the issue of whether Ryle's argument is a sound one. Grayling (1995, p. 50) mentions in effect that a sceptic critic could object to Ryle's polar concept argument that a same line of reasoning applied to other dualities such as 'perfect/imperfect', 'mortal/immortal', 'finite/infinite' would entail the existence of perfect, immortal or infinite entities. An objection along the same lines could then be raised against the present argument for abstracta. However, in the present context, the above three dualities do not deserve the same type of response. For that reason, I shall consider them in turn. Begin then with the perfect/imperfect duality. From the above, it should be apparent that the perfect/imperfect duality does not fall under the scope of the present argument. For perfect is not a simple quality. In effect, perfect can be defined as the sum of all simple positive qualities. Thus, perfect can be characterised as a complex or composite quality. But as we have seen, the scope of the present argument is restricted to simple qualities. For that reason, the existence of perfect entities is not entailed by the present argument.

  Let us turn now to the mortal/immortal duality. At this step, it should be pointed out that it is not clear whether mortal can be considered as a simple quality, in the sense defined above. For that reason, I shall replace it by the temporal/atemporal duality. This latter pair is made up of two conceptual polarities that can be regarded unambiguously as simple qualities. Now it should be acknowledged that the temporal/atemporal duality also falls under the scope of the above argument. And a same line of reasoning yields the existence of atemporal entities. I shall endorse such a consequence here. In effect, the present argument is also for the existence of atemporal entities. But is there something counter-intuitive here? It seems that some objects such as numbers are obvious candidates for this definition. In this context, natural numbers can be considered consistently as both abstract and atemporal entities.

  Now the same goes for the application of the present argument to the infinite/finite duality. For it should be acknowledged that infinite and finite are simple qualities in the sense defined above. Thus the argument also applies to these latter concepts and entails the existence of infinite objects. But such inference should not be very disturbing, I think, for it is much in line with our current intuitions. Just as in the previous case regarding the temporal/atemporal duality, there exist immediate candidates for the definition of infinite entities. Natural numbers, for example, straightforwardly instantiate the property of being both abstract and infinite.

  Let us consider, second, another line of objection. It could be opposed to the present argument that a similar line of reasoning leads to the existence of *impossible* objects. Few would doubt, in effect, that we currently have a large body of evidence in favour of the existence of *possible* objects. Hence, according to the above argument, from the possible/impossible duality, we can infer the existence of impossible objects. But as this latter notion is self-contradictory, the objection goes, the whole enterprise is vowed to inconsistency. However, this line of objection does not undermine the force of the argument, I think. For the present argument is only concerned with instantiable objects. It begins with the observation that many objects exemplifying both poles of a given duality do exist. It pursues by inferring the existence of objects that instantiate a pole of the abstract/concrete duality. But in all cases, the present argument is only concerned with pairs of polar contraries that are compatible with existence. Perhaps, some would agree that certain objects possess the property of being impossible, contradictory or imaginary. But such inferences don't fall under the scope of the present argument. For

the dualities which are concerned with the present argument need at least to be *instantiable*. As a consequence, predicates such as *impossible*, *inexistent*, *imaginary*, *contradictory*, *inconceivable*, etc., should be discarded from the beginning. And all non-instantiable dualities (i.e. dualities which contain at least one non-instantiable pole) such as possible/impossible, existent/inexistent, coherent/incoherent, etc., are not targeted by the current argument. In addition, the same response prevails for a similar line of objection that would respectively infer the existence of *inexistent*, *inconceivable*, *imaginary* objects, from the existent/inexistent, conceivable/inconceivable, real/imaginary dualities.

It is worth examining, third, a different line of objection. It runs as follows. The present argument rests on the necessity of instantiating both poles of all dualities. But it could be retorted that certain poles of some dualities need not being instantiated. And such is the case, the objection runs, for the abstract pole of the abstract/concrete duality. It should be apparent that this latter objection challenges premise (4), according to which, due to the well-balance requirement of our universe, all instantiable polar dualities are exemplified. But this latter objection is not very promising, I think. For the present polar concept argument is concerned with our universe's well-balanced requirement. And this well-balance results precisely from the instantiation of both poles of each duality. Consider the case of the bright/dark duality. Imagine our universe containing only dark objects, with all bright objects missing. Would we expect to find ourselves in such a universe? No. For the emergence of carbon-based life would be impossible in such one-sided (from the viewpoint of the dark/bright duality) universe. Or consider alternatively the situation if all objects in our universe were static and no objects were dynamic. Or else imagine if our present universe only contained cold objects, and were entirely devoid of hot ones. All such universes would be very unfriendly, to say the least. Now the same applies to the abstract pole of the abstract/concrete duality. Perhaps it could be helpful to recall, at this step, one major premise of the fine-tuning argument. From the fact that many cosmological constants are fine-tuned for the emergence of carbon-based life, the fine-tuning argument derives the conclusion that this latter feature of our universe is non-random and due to the intention of its Creator. Now setting aside the controversial conclusion of the fine-tuning argument, it appears that the premise according to which the cosmological constants are fine-tuned for further emergence of carbon-based life can be replaced by the two following steps:

(1f)   the cosmological constants of our universe are fine-tuned for the emergence of carbon-based life

(2f)   ∴ the cosmological constants of our universe are fine-tuned for the instantiation   from (1f)
          of the following dualities: large/small, positive/negative, external/internal, bright/dark, static/dynamic, visible/invisible, cold/hot, etc.

At this point, it is worth noting that the assertion according to which our universe is well-balanced is even weaker than the uncontroversial premise (1f) of the fine-tuning argument. For consider the following instance of anthropic coincidence related to the *gravitational force constant* which is part of (1f):[5]

(3f)   if the gravitational force constant had been larger then stars would be have been too *hot* to allow for carbon-based life chemistry[6]

(4f)   if the gravitational force constant had been smaller then stars would have been be too *cool* to permit carbon-based life chemistry

Now it appears that these two propositions can be recast as follows:

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[5] From Ross (1999).
[6] Paraphrasing Ross.

(5f)   if the gravitational force constant had been larger then the *cold* pole of the cold/hot duality would have not been instantiated

(6f)   if the gravitational force constant had been smaller then the *hot* pole of the cold/hot duality would have not been instantiated

To take another example:

(7f)   if the velocity of light had been faster then stars would have been be too luminous for life support

(8f)   if the velocity of light had been slower then stars would have been insufficiently luminous for life support

can be restated into the weaker:

(9f)   if the velocity of light had been faster then the *dark* pole of the bright/dark duality would have not been exemplified

(10f) if the velocity of light had been slower then the *bright* pole of the bright/dark duality would have not been exemplified

More generally, every anthropic coincidence can be restated into the weaker couple of propositions:

(11f) if the <cosmological constant> had been larger then the A pole of the A/Ā duality would have not been instantiated

(12f) if the <cosmological constant> had been smaller then the Ā pole of the A/Ā duality would have not been instantiated

At this step, it should be apparent that challenging (2f) also implies being committed to doubt (1f), while on the other hand it is a widely accepted premise of the fine-tuning argument.

The above argument could be attacked, fourth, on the grounds that is not deductive, but rather inductive. According to this line of objection, the present argument is a disguised inductive argument. If the argument were inductive instead of deductive, it would be probabilistic and as such, its impact would be weaker than in its original deductive presentation. As mentioned above, it should be acknowledged that the argument can be presented alternatively as an inductive argument. The inductive form of the argument begins with an enumeration of all exemplified polar dualities. From this, it derives a generalisation to all polar dualities. The conclusion that the abstract/concrete duality and in particular its abstract pole is also instantiated ensues. If the above argument is to be considered as essentially inductive, this has the effect of weakening the argument by making it inductive rather than deductive. However, the present argument is not intended to count as a proof yielding absolute certainty. Then choose whatever variation - deductive or inductive - of the argument you prefer. In either case, the essence of the argument remains in force.

It would also be tempting to challenge, fifth, premise (1), namely the fact that our universe is well-balanced. But such an objection is not very promising, I think. In effect, our universe is about 14 billion years old. How could our universe have lasted so long if it hadn't been well-balanced? Considering now its spatial extension, a question of the same nature arises. For our universe extends billions of light years in any directions. How could our universe have occupied such huge spatial region without being well-balanced? And again: How could our universe contain so much objects such as neutrons, monkeys, stars, galaxies, etc., and a total number of atoms amounting to 10^{80}, without being well-balanced?

Another line of objection that could be pressed, sixth, against the present argument is that it is simply a generalisation of Ryle's argument. Although both arguments share a common ground, I shall stress that Ryle's argument is slightly differently motivated. In effect, the key concept in the current polar concept argument is the well-balanced property. A key premise is in effect that being well-balanced is

a prominent feature of our universe. And this last premise is reinforced by the additional premise based on the empirical fact that some properties of our universe currently instantiate this well-balanced property. Thus, the present argument is not entirely a priori as could be characterised Ryle's argument. The present argument incorporates in effect some empirical features of our universe. In addition, I should be stressed that Ryle's argument contrasts 'error' and 'getting it right'. But it worth emphasizing that 'error' has a pejorative connotation and 'getting it right' reveals a meliorative nuance. Hence, 'error' is a negative concept and 'getting it right' is a positive one. By contrast, the current argument is only concerned with neutral concepts and pairs of neutral opposites. Consequently, it is worth stressing that the present argument would be inapplicable to the 'error/getting it right' pair of concepts.

In conclusion, it is worth stating accurately the scope of the above argument. It is not designed in effect to serve as a proof of the existence of abstracta.[7] For given our current high standards, it should be acknowledged that it does not meet our present criteria of proof. Rather, the present argument aims at reinforcing an initial credence that abstract objects could exist. The above argument is simply designed to increase our a priori belief about the existence of abstracta. As such, it is consistent with the Quine-Putnam indispensability argument. It is also consistent with recent trends in cosmology and in particular with the level IV of the kind of multiverse described in Tegmark (2003). In this context, the purpose of the present argument for abstracta is to provide some additional grounds in support of the hypothesis that abstract objects do exist.

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[7] This raises the interesting question of what could count (if any) as a proof of the existence of abstracta. Or alternatively put: Is the existence of abstracta a testable hypothesis?