Generalized Continuum Hypothesis and the Axiom of Combinatorial Sets

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Prologue

In an earlier paper [1], intuitive set theory (IST) was defined as the theory we get when we add the two axioms, *monotonicity* and *fusion*, to ZF theory. Here we attempt to replace the axiom of monotonicity with a simpler axiom we call, *axiom of combinatorial sets*. 
Axiom of Combinatorial Sets

If $k$ is an ordinal, we write $\binom{\mathfrak{N}_\alpha}{k}$ for the cardinality of the set of all subsets of $\mathfrak{N}_\alpha$ with cardinality of $k$.

\[\mathfrak{N}_{\alpha+1} = \binom{\mathfrak{N}_\alpha}{\mathfrak{N}_\alpha}.\]
Derivation

We derive the Generalized Continuum Hypothesis from the axiom of combinatorial sets as below:

\[ 2^{\aleph_\alpha} = \binom{\aleph_\alpha}{0} + \binom{\aleph_\alpha}{1} + \binom{\aleph_\alpha}{2} + \cdots + \binom{\aleph_\alpha}{\aleph_0} + \cdots \binom{\aleph_\alpha}{\aleph_\alpha}. \]

Note that \( \binom{\aleph_\alpha}{1} = \aleph_\alpha \). Since, there are \( \aleph_\alpha \) terms in this addition and \( \binom{\aleph_\alpha}{k} \) is a monotonically nondecreasing function of \( k \), we can conclude that

\[ 2^{\aleph_\alpha} = \binom{\aleph_\alpha}{\aleph_\alpha}. \]

Using axiom of combinatorial sets, we get

\[ 2^{\aleph_\alpha} = \aleph_{\alpha+1}. \]
Epilogue

In view of the fact that we can derive the generalized continuum hypothesis from the axiom of combinatorial sets, we can replace the axiom of monotonicity \([1, 2]\) with the axiom of combinatorial sets, in the definition of intuitive set theory.
References


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